جامعة تكريت ـ كلية التربية للبنات ـ قسم الرياضيات

المرحلة الثانية ـ المعادلات التفاضلية الاعتيادية

الفصل التمهيدي ـ طرائق التكامل

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عنوان المحاضرة :

الطريقة الثامنة : التكامل المجدول

الطريقة الثامنة   Eighth Method

التكامل المجدول   Tabular Integration

هذه الطريقة مختصرة لطريقة (udv) وهي تقوم على أساس اختيار دالة ($f(x)$) يمكن اشتقاقها عدد

من المرات إلى تصــبح صــفرا واختيار الدالة الأخرى ($g(x)$) بحيث يمكن أن نكاملها بعدد مرات اشــتقاق

الدالة الأولى

Example (47): By Tabular Integration Evaluate $I = \int x^2 e^x dx$

Sign الاشارة	$f(x)$ and its derivatives الدالة $f(x)$ ومشتقاتها	$g(x)$ and its integrals الدالة $g(x)$ وتكاملاتها
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
$-$	0	e^x

$$I = \int x^2 e^x dx = +x^2 e^x - 2xe^x + 2e^x + c$$

Example (48): By Tabular Integration Evaluate $I = \int x^3 \sin x dx$

sign	$f(x)$	$g(x)$
$+$	x^3	$\sin x$
$-$	$3x^2$	$-\cos x$
$+$	$6x$	$-\sin x$
$-$	6	$\cos x$
$+$	0	$\sin x$

$$I = \int x^3 \sin x dx = x^3(-\cos x) - 3x^2(-\sin x) + 6x\cos x - 6\sin x + c$$

$$I = -x^3 \cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + c$$

Exercise (5-1): Evaluate the integrals use integration by part (*udv*) or tabular integration method.   Exe. (1-12) page- 389 – questions(1-22) blue book

No.	Question	Answer
1	$\int x^3 \cos x\,dx$	$I = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + c$
2	$\int x \cosh x\,dx$	$I = x \sinh x - \cosh x + c$
3	$\int x^2 \ln x\,dx$	$I = \dfrac{x^3}{3} \ln x - \dfrac{x^3}{9} + c$
4	$\int \dfrac{x\,dx}{\sin^2 x}$	$I = -x \cot x + \ln|\sin x| + c$
5	$\int x^3 \ln x\,dx$	$I = \dfrac{x^4}{4} \ln x - \dfrac{x^4}{16} + c$
6	$\int e^{ax} \cos bx\,dx$	$I = \dfrac{b^2}{a^2 + b^2}\left(\dfrac{1}{b} e^{ax} \sin bx + \dfrac{a}{b^2} e^{ax} \cos bx\right) + c$
7	$\int x \sin 3x\,dx$	$I = \dfrac{-x}{3} \cos 3x + \dfrac{1}{9} \sin 3x + c$
8	$\int \sin(\ln x)\,dx$	$I = \dfrac{-x \cos(\ln x) + x \sin(\ln x)}{2} + c$
9	$I = \int x^3 e^x\,dx$	$I = \int x^3 e^x\,dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6 e^x + c$
10	$\int \ln(a^2 + x^2)\,dx$	$I = x \ln(a^2 + x^2) - 2x + 2a \tan^{-1} \dfrac{x}{a} + c$
11	$\int x^2 \cos(2x)\,dx$	$I = \dfrac{1}{2} x^2 \sin 2x + \dfrac{1}{2} x \cos 2x - \dfrac{1}{4} \sin 2x + c$
12	$\int x\sqrt{1+x}\,dx$	$I = \dfrac{2x}{3}(1+x)^{3/2} - \dfrac{4}{15}(1+x)^{5/2} + c$ $I = \dfrac{2}{5}(1+x)^{5/2} - \dfrac{2}{3}(1+x)^{3/2} + c$
13	$\int x^3 \sqrt{x-1}\,dx$	$I = \dfrac{2}{3} x^3 (x-1)^{3/2} - \dfrac{18}{5} x^2 (x-1)^{5/2} + \dfrac{48}{105} x(x-1)^{7/2} - \dfrac{96}{945}(x-1)^{9/2} + c$ $I = \dfrac{2}{9}(x-1)^{9/2} + \dfrac{6}{7}(x-1)^{7/2} + \dfrac{6}{5}(x-1)^{5/2} + \dfrac{2}{3}(x-1)^{3/2} + c$
14	$\int \sin^2 x\,dx$	$I = \dfrac{1}{2} x - \dfrac{1}{4} \sin 2x + c$
15	$\int \sin x \sinh x\,dx$	$I = \dfrac{\sin x \cosh x - \cos x \sinh x}{2} + c$
16	$\int \sec^2 x\,dx$	$I = \int (\sec x)^2\,dx = \dfrac{(\sec x)^0 (\tan x)}{1} + \dfrac{0}{1}\int (\sec x)^0\,dx = \tan x + c$
17	$\int \ln(x^2)\,dx$	$I = x \ln(x^2) - 2x + c$

18	$\int x\tan^{-1}xdx$	$I=\dfrac{1}{2}x^2\tan^{-1}x-\dfrac{1}{2}\left(x-\tan^{-1}x+c\right)$
19	$\int \dfrac{\ln x}{x^3}dx$	$I=\dfrac{-1}{2x^2}\ln x-\dfrac{1}{4x^2}+c$
20	$\int \dfrac{xe^x}{(1+x)^2}dx$	$I=xe^x\left(\dfrac{-1}{1+x}\right)+e^x+c$
21	$\int \dfrac{d^3y}{dx^3}ydx$	

Exercise (5-2): Evaluate the integrals use integration by part (*udv*) or tabular integration method.

No.	Question	Answer
1	$\int x^3\cos(3x)dx$	$I=\dfrac{1}{3}x^3\sin 3x+\dfrac{1}{3}x^2\cos 3x-\dfrac{2}{9}x\sin 3x-\dfrac{2}{27}\cos 3x+c$
2	$\int \cot^{-1}xdx$	$I=x\cot^{-1}x+\dfrac{1}{2}\ln\left|1+x^2\right|+c$
3	$\int x^2e^{-x}dx$	$I=\int x^2e^{-x}dx=-x^2e^{-x}-2xe^{-x}-2e^{-x}+c$
4	$\int (x+2)e^{ax}dx$	$I=\dfrac{1}{a}(x+2)(e^{ax})-\dfrac{1}{a^2}e^{ax}+c$
5	**a.** $\int \ln(x+1)dx$ **b.** $\int x\ln(x+1)dx$	**a.** $I=x\ln(x+1)-x+\ln(x+1)+c$ **b.** $I=\dfrac{x^2}{2}\ln(x+1)-\dfrac{1}{2}\left(\dfrac{x^2}{2}-x\right)-\dfrac{1}{2}\ln(x+1)+c$
6	$\int x^2\ln 3xdx$	$I=\dfrac{x^3}{3}\cdot(\ln 3x)-\dfrac{1}{9}x^3+c$
7	$\int \cos(\ln x)dx$	$I=\dfrac{x\sin(\ln x)+x\cos(\ln x)}{2}+c$
8	$\int x\sec^2dx$	$I=x\tan x+\ln\left|\cos x\right|+c$
9	$\int x\sin^{-1}xdx$	$I=\dfrac{x^2}{2}\sin^{-1}x+\dfrac{1}{4}\cos^{-1}x+\dfrac{1}{4}x\sqrt{1-x^2}+c$

Exercise (5-3): Evaluate the integrals use integration by part (*udv*) or tabular integration method

No.	Question	Answer
Integration by Parts		
Evaluate the integrals in Exercises 1-24 using integmtion by parts (*udv*) method		
1	$\int x\sin\dfrac{x}{2}dx$	$I=-2x\cos\dfrac{x}{2}+4\sin\dfrac{x}{2}+c$

45

2	$\int \theta \cos \pi \theta d\theta$	$I = \dfrac{\theta}{\pi} \sin \pi\theta - \dfrac{1}{\pi^2} \cos \pi\theta + c$
3	$\int t^2 \cos t dt$	$I = t^2 \sin t + 2t \cos t - 2\sin t + c$
4	$\int x^2 \sin x dx$	$I = -x^2 \cos x + 2x \sin x + 2\cos x + c$
5	$\int x \ln x dx$	$I = \dfrac{x^2}{2} \ln x - \dfrac{1}{4} x^2 + c$
6	$\int x^3 \ln x dx$	$I = \dfrac{x^4}{4} \ln x - \dfrac{x^4}{16} + c$
7	$\int x e^{2x} dx$	$I = \dfrac{2}{4} x e^{2x} - \dfrac{1}{4} e^{2x} + c$
8	$\int x e^{3x} dx$	$I = \dfrac{1}{3} x e^{3x} - \dfrac{1}{9} e^{3x} + c$
9	$\int x^2 e^{-x} dx$	$I = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$
10	$\int (x^2 - 2x + 1) e^{2x} dx$	$I = \dfrac{1}{2}(x^2 - 2x + 1)e^{2x} - \dfrac{1}{4}(2x - 2)e^{2x} + \dfrac{1}{4} e^{2x} + c$
11	$\int \tan^{-1} 2x dx$	$I = x \tan^{-1} 2x - \dfrac{1}{4} \ln\left|4x^2 + 1\right| + c$
12	$\int \sin^{-1} 3x dx$	$I = x \sin^{-1} 2x + \dfrac{1}{2} \sqrt{1 - 4x^2} + c$
13	$\int x \sec^2 x dx$	$I = x \tan x + \ln\left|\cos x\right| + c$
14	$\int 4x \sec^2 2x dx$	$I = 2x \tan 2x + \ln\left|\cos 2x\right| + c$
15	$\int x^3 e^x dx$	$I = \int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$
16	$\int x^4 e^{-x} dx$	$I = -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24e^{-x} + c$
17	$\int (x^2 - 5x) e^x dx$	$I = (x^2 - 5x)e^x - (2x - 5)e^x + 2e^x + c$
18	$\int (x^2 + x + 1) e^x dx$	$I = (x^2 + x + 1)e^x - (2x + 1)e^x + 2e^x + c$
19	$\int x^5 e^x dx$	$I = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120e^x + c$
20	$\int x^2 e^{4x} dx$	$I = \dfrac{1}{4} x^2 e^{4x} - \dfrac{1}{8} x e^{4x} + \dfrac{1}{32} e^{4x} + c$
21	$\int e^{-x} \sin x dx$	$I = \dfrac{e^{-x} \sin x + e^{-x} \cos x}{-2} + c$
22	$\int e^{-x} \cos x dx$	$I = \dfrac{e^{-x} \sin x - e^{-x} \cos x}{2} + c$
23	$\int e^{2x} \cos 3x dx$	$I = \dfrac{9}{13}\left(\dfrac{1}{3} e^{2x} \sin 3x + \dfrac{2}{9} e^{2x} \cos 3x\right) + c$
24	$\int e^{-2x} \sin 2x dx$	$I = \dfrac{-1}{4} e^{-2x} \cos 2x - \dfrac{1}{4} e^{-2x} \sin 2x + c$
Using Substitution		

Evaluate the integrals in Exercises 25-30 by using a substitutioo prior to integration by parts.

25	$\int e^{\sqrt{3x+9}}dx$	$I = \dfrac{2}{3}(\sqrt{3x+9} \cdot e^{\sqrt{3x+9}} - e^{\sqrt{3x+9}}) + c$
26	$\int x\sqrt{1-x}dx$	$I = -\dfrac{2}{3}(1-x)^{3/2} + \dfrac{2}{5}(1-x)^{5/2} + c$
27	$\int x\tan^2 x dx$	$I = -\dfrac{x^2}{2} + x\tan x + \ln|\cos x| + c$
28	$\int \ln(x + x^2)dx$	$I = x\ln(x + x^2) - 2x + \ln|1 + x| + c$
29	$\int \sin(\ln x)dx$	$I = \dfrac{-x\cos(\ln x) + x\sin(\ln x)}{2} + c$
30	$\int x(\ln x)^2 dx$	$I = \dfrac{x^2}{2}(\ln x)^2 - \dfrac{x^2}{2}\ln x + \dfrac{x^2}{4} + c$

Evaluating Integrals

Evaluate the integrals in Exercises 31-50. Some integrals do not require integration by parts.

31	$\int x \cdot \sec x^2 dx$	$I = \dfrac{1}{2}\ln|\sec x^2 + \tan x^2| + c$
32	$\int \dfrac{\cos\sqrt{x}}{\sqrt{x}}dx$	$I = 2\sin\sqrt{x} + c$
33	$\int x(\ln x)^2 dx$	$I = \dfrac{1}{2}(\ln x)^2 e^{2(\ln x)} - \dfrac{1}{2}(\ln x)e^{2(\ln x)} + \dfrac{1}{4}e^{2(\ln x)} + c$
34	$\int \dfrac{dx}{x(\ln x)^2}$	$I = \dfrac{-1}{\ln x} + c$
35	$\int \dfrac{\ln x}{x^2}dx$	$I = \dfrac{-\ln x}{x} - \dfrac{1}{x} + c$
36	$\int \dfrac{(\ln x)^3}{x}dx$	$I = \dfrac{(\ln x)^4}{4} + c$
37	$\int x^3 e^{x^4} dx$	$I = \dfrac{1}{4}e^{x^4} + c$
38	$\int x^5 e^{x^3} dx$	$I = \dfrac{1}{3}x^3 e^{x^3} - \dfrac{1}{3}e^{x^3} + c$
39	$\int x^3\sqrt{x^2+1}dx$	$I = \dfrac{1}{3}x^2(x^2+1)^{3/2} - \dfrac{2}{15}(x^2+1)^{5/2} + c$
40	$\int x^2 \cdot \sin x^3 dx$	$I = -\dfrac{1}{3}\cos x^3 + c$
41	$\int \sin 3x\cos 2x dx$	$I = -\dfrac{2}{5}\sin 3x\sin 2x - \dfrac{3}{5}\cos 3x\cos 2x + c$
42	$\int \sin 2x\cos 4x dx$	$I = \dfrac{1}{3}\sin 2x\sin 4x + \dfrac{1}{6}\cos 2x\cos 4x + c$
43	$\int e^x \sin e^x dx$	$I = -\cos e^x + c$

47

44	$\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$	$I = 2e^{\sqrt{x}} + c$
45	$\displaystyle\int \cos\sqrt{x}\,dx$	$I = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x}$
46	$\displaystyle\int \sqrt{x}\,e^{\sqrt{x}}dx$	$I = 2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} + c$
47	$\displaystyle\int_{0}^{2\pi} \theta^2 \sin 2\theta\, d\theta$	$\dfrac{\pi^2 - 4}{8}$
48	$\displaystyle\int_{0}^{2\pi} x^3 \cos 2x\, dx$	$\dfrac{3(4 - \pi^2)}{16}$
49	$\displaystyle\int_{2\sqrt{3}}^{2} t\,\sec^{-1} t\, dt$	$\dfrac{5\pi - 3\sqrt{3}}{9}$
50	$\displaystyle\int_{0}^{1/\sqrt{2}} 2x\sin^{-1}(x^2)\, d\theta$	$\dfrac{\pi + 6\sqrt{3} - 12}{12}$