جامعة تكريت – كلية التربية للبنات – قسم الرياضيات

المرحلة الثانية – المعادلات التفاضلية الاعتيادية

الفصل التمهيدي – طرائق التكامل

أ.د. عامر فاضل نصار
amer6767@tu.edu.iq

عنوان المحاضرة :

الطريقة التاسعة : حذف الجذر

الطريقة العاشرة : طريقة تكامل دوال من النوع $(\sin x)^n$, $(\cos x)^n$

الطريقة التاسعة Ninth Method

طريقة حذف الجذر التربيعي

Example (49): Evaluate $I=\int\sqrt{1+\cos 4x}\ dx$

$$\cos^2 x=\frac{1+\cos 2x}{2}\quad\Rightarrow\quad \cos^2 2x=\frac{1+\cos 4x}{2}\quad\Rightarrow\quad 2\cos^2 2x=1+\cos 4x$$

$$I=\int\sqrt{1+\cos 4x}\ dx=\int\sqrt{2\cos^2 2x}\ dx=\sqrt{2}\int\cos 2x\ dx=\frac{\sqrt{2}}{2}\int\cos 2x\cdot(2)dx=\frac{1}{\sqrt{2}}\sin 2x+c$$

Example (50): Evaluate $I=\int\sqrt{1-\cos x}\ dx$

سنحل هذا المثال بطريقتين

- $\sin^2 x=\frac{1-\cos 2x}{2}\quad\Rightarrow\quad \sin^2(\frac{x}{2})=\frac{1-\cos x}{2}\quad\Rightarrow\quad 2\sin^2(\frac{x}{2})=1-\cos x$

$$I=\int\sqrt{1-\cos x}\ dx=\int\sqrt{2\sin^2(\frac{x}{2})}\ dx=\sqrt{2}\int\sin(\frac{x}{2})\ dx=2\sqrt{2}\int\sin(\frac{x}{2})\,(\frac{1}{2})\,dx=-2\sqrt{2}\cos(\frac{x}{2})+\mathrm{c}$$

- $I=\int\sqrt{1-\cos x}\ dx=\int\sqrt{1-\cos x}\cdot\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}dx=\int\frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1+\cos x}}dx=\int\frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}}dx$

$$I=\int\frac{\sqrt{\sin^2 x}}{\sqrt{1+\cos x}}dx=\int\frac{\sin x}{\sqrt{1+\cos x}}dx=\int(1+\cos x)^{-1/2}\sin x dx=\frac{(1+\cos x)^{1/2}}{1/2}+c$$

$$I=2\sqrt{1+\cos x}+c$$

الطريقة العاشرة Tenth Method

طريقة تكامل دوال من النوع $(\sin x)^n$, $(\cos x)^n$

$$\int(\sin x)^n dx=\int(\sin x)^{n-1}(\sin x)^1 dx$$

$$u=(\sin x)^{n-1}\quad\Rightarrow du=(n-1)(\sin x)^{n-2}\cos x dx\quad\&\quad dv=\sin x dx\quad\Rightarrow\quad v=-\cos x$$

$$\int(\sin x)^n dx=uv-\int v du=(\sin x)^{n-1}(-\cos x)-\int(-\cos x)\,(n-1)(\sin x)^{n-2}\cos x dx$$

$$\int(\sin x)^n dx=-\cos x(\sin x)^{n-1}+(n-1)\int(\sin x)^{n-2}(\cos x)^2 dx$$

$$\int(\sin x)^n dx=-\cos x(\sin x)^{n-1}+(n-1)\int(\sin x)^{n-2}(1-\sin^2 x)dx$$

$$\int(\sin x)^n dx=-\cos x(\sin x)^{n-1}+(n-1)\int[(\sin x)^{n-2}-\sin^2 x]dx$$

$$\int(\sin x)^n dx=-\cos x(\sin x)^{n-1}+(n-1)\int(\sin x)^{n-2}dx-(n-1)\int(\sin x)^n dx$$

$$\int (\sin x)^n dx + (n-1)\int (\sin x)^n dx = -\cos x(\sin x)^{n-1} + (n-1)\int (\sin x)^{n-2} dx$$

$$n\int (\sin x)^n dx = -\cos x(\sin x)^{n-1} + (n-1)\int (\sin x)^{n-2} dx$$

$$\int (\sin x)^n dx = \frac{-(\sin x)^{n-1}\cos x}{n} + \frac{n-1}{n}\int (\sin x)^{n-2} dx$$

وبنفس الطريقة تماما نشتق القانون

$$\int (\cos x)^n dx = \frac{(\cos x)^{n-1}\sin x}{n} + \frac{n-1}{n}\int (\cos x)^{n-2} dx$$

اما تكامل دوال من النوع $(\sin ax)^n$, $(\cos ax)^n$

$$\int (\sin ax)^n dx = \int (\sin ax)^{n-1} (\sin ax)^1 dx$$

$$u = (\sin ax)^{n-1} \quad \Rightarrow du = a\,(n-1)(\sin ax)^{n-2}\cos ax dx \quad \& \quad dv = \sin ax dx \quad \Rightarrow \quad v = \frac{-1}{a}\cos ax$$

$$\int (\sin x)^n dx = uv - \int vdu = (\sin ax)^{n-1}(\frac{-1}{\text{a}}\cos ax) - \int (\frac{-1}{\text{a}}\cos x)\,(\text{a})(n-1)(\sin ax)^{n-2}\cos ax dx$$

$$\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int (\sin ax)^{n-2}\,(\cos ax)^2 dx$$

$$\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int (\sin ax)^{n-2}\,(1-\sin^2 ax)dx$$

$$\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int [(\sin ax)^{n-2} - \sin^2 ax]dx$$

$$\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int (\sin ax)^{n-2} dx - (n-1)\int (\sin ax)^n dx$$

$$\int (\sin ax)^n dx + (n-1)\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int (\sin ax)^{n-2} dx$$

$$n\int (\sin ax)^n dx = \frac{-1}{\text{a}}(\cos ax)(\sin ax)^{n-1} + (n-1)\int (\sin ax)^{n-2} dx$$

$$\int (\sin ax)^n dx = \frac{-(\sin ax)^{n-1}\cos ax}{an} + \frac{n-1}{n}\int (\sin ax)^{n-2} dx$$

وبنفس الطريقة تماما نشتق القانون

$$\int (\cos ax)^n dx = \frac{(\cos ax)^{n-1}\sin ax}{an} + \frac{n-1}{n}\int (\cos ax)^{n-2} dx$$

Example (51): Evaluate $I = \int \sin^3 x dx$

$$\int (\sin x)^n dx = \frac{-(\sin x)^{n-1}\cos x}{n} + \frac{n-1}{n}\int (\sin x)^{n-2} dx$$

$$I = \int \sin^3 x dx = \frac{-(\sin x)^{3-1}\cos x}{3} + \frac{3-1}{3}\int (\sin x)^{3-2} dx = \frac{-(\sin x)^2\cos x}{3} + \frac{2}{3}\int \sin x dx$$

$$I = \frac{-(\sin x)^2\cos x}{3} + \frac{2}{3}\int \sin x dx = \frac{-(\sin x)^2\cos x}{3} + \frac{2}{3}(-\cos x) + c$$

Example (52): Evaluate $I = \int \cos^5 x dx$

$$\int (\cos x)^n dx = \frac{(\cos x)^{n-1}\sin x}{n} + \frac{n-1}{n}\int (\cos x)^{n-2} dx$$

$$I = \int \cos^5 x dx = \frac{(\cos x)^{5-1}\sin x}{5} + \frac{5-1}{5}\int (\cos x)^{5-2} dx = \frac{(\cos x)^4\sin x}{5} + \frac{4}{5}\int (\cos x)^3 dx$$

$$I = \frac{(\cos x)^4\sin x}{5} + \frac{4}{5}\left(\frac{(\cos x)^2\sin x}{3} + \frac{2}{3}\int \cos x dx\right)$$

$$I = \frac{(\cos x)^4\sin x}{5} + \frac{4(\cos x)^2\sin x}{15} + \frac{8}{15}\sin x + c$$