جامعة تكريت – كلية التربية للبنات – قسم الرياضيات

المرحلة الثانية – المعادلات التفاضلية الاعتيادية

الفصل التمهيدي – طرائق التكامل

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عنوان المحاضرة :

الطريقة الثالثة عشر : طريقة تكاملات من الأنواع

$$\int \cos(mx)\cos(nx)dx \text{ ، } \int \sin(mx)\cos(nx)dx \text{ ، } \int \sin(mx)\sin(nx)dx$$

الطريقة الثالثة عشر Thirteenth method

طريقة تكاملات من الأنواع $\int \sin(mx)\sin(nx)dx$ ، $\int \sin(mx)\cos(nx)dx$ ، $\int \cos(mx)\cos(nx)dx$

نحتاج في هذه الطريقة الى القوانين السابقة الآتية

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$

ومن هذه العلاقات يمكن استنتاج

$$\sin(mx)\sin(nx)=\frac{1}{2}[\cos(m-n)x-\cos(m+n)x]$$

$$\cos(mx)\cos(nx)=\frac{1}{2}[\cos(m-n)x+\cos(m+n)x]$$

$$\sin(mx)\cos(nx)=\frac{1}{2}[\sin(m-n)x+\sin(m+n)x]$$

Example (56): Evaluate $\int \sin(3x)\cos(5x)dx$

$$\sin(mx)\cos(nx)=\frac{1}{2}[\sin(m-n)x+\sin(m+n)x]$$

$$\int \sin(3x)\cos(5x)dx=\frac{1}{2}\int[\sin(3-5)x+\sin(3+5)x]dx=\frac{1}{2}\int[\sin(-2x)+\sin(8x)]dx$$

$$=\frac{1}{2}[\frac{-1}{2}\int \sin(-2x)(-2)dx+\frac{1}{8}\int \sin(8x)(8)dx]$$

$$=(1/4)\cos(-2x)-(1/16)\cos(8x)+c$$

Example (57): Evaluate $\int \sin(x)\sin(6x)dx$

$$\sin(mx)\sin(nx)=\frac{1}{2}[\cos(m-n)x-\cos(m+n)x]$$

$$\int \sin(x)\sin(6x)dx=\frac{1}{2}\int[\cos(1-6)x-\cos(1+6)x]dx=\frac{1}{2}\int[\cos(-5x)-\cos(7x)]dx$$

$$=\frac{1}{2}[\frac{-1}{5}\int \cos(-5x)(-5)dx-\frac{1}{7}\int \cos(7x)(7)dx]$$

$$=\frac{-1}{10}\sin(-5x)-\frac{1}{14}\sin(7x)+c=\frac{1}{10}\sin(5x)-\frac{1}{14}\sin(7x)+c$$

Example (58): Evaluate $\int \cos(3x)\cos(x)dx$

$$\cos(mx)\cos(nx)=\frac{1}{2}[\cos(m-n)x+\cos(m+n)x]$$

$$\int\cos(3x)\cos(x)dx=\frac{1}{2}\int[\cos(2x)+\cos(4x)]dx$$

$$=\frac{1}{2}[\frac{1}{2}\int\cos(2x)(2)dx+\frac{1}{4}\int\cos(4x)(4)dx]$$

$$=\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)+c$$

الطريقة الرابعة عشر Fourteenth method

طريقة تكاملات من الأنواع $\int(\sin x)^m(\cos x)^n dx$

هناك عدة احتمالات لقيم m و n

الاحتمال الأول : إذا كان $n=0$ أو $m=0$ فان الحل سيكون حسب الطريقة الثامنة

الاحتمال الثاني : إذا كان احدهما عدد فردي وليكن m هو العدد الفردي

$$(\sin x)^m=(\sin x)^{2k+1}=(\sin^2 x)^k(\sin x)=(1-\cos^2 x)^k(\sin x)$$

ونعوض هذه الفرضية في السؤال المراد حله فنتمكن من حل السؤال

الاحتمال الثالث : إذا كان كل من m و n أعداد زوجية فنستخدم المتطابقتين الاتيتين

$$(\cos x)^2=\frac{1+\cos 2x}{2} \qquad \& \qquad (\sin x)^2=\frac{1-\cos 2x}{2}$$

صيغ التخفيض

يمكن أحيانًا تقصير الوقت المطلوب لعمليات التكامل حسب طريقة التكامل بالأجزاء (udv) عن طريق تطبيق صيغ التخفيض مثل

$$\int(\ln x)^n dx=x(\ln x)^n-n\int(\ln x)^{n-1}dx$$

$$\int\sin^n x\cdot\cos^m xdx=-\frac{\sin^{n-1}x\cdot\cos^{m+1}x}{m+n}+\frac{n-1}{m+n}\int\sin^{n-2}x\cdot\cos^m xdx\ ,\ n\neq -m \quad \text{reduces } \sin^n x$$

$$\int\sin^n x\cdot\cos^m xdx=\frac{\sin^{n+1}x\cdot\cos^{m-1}x}{m+n}+\frac{m-1}{m+n}\int\sin^n x\cdot\cos^{m-2}xdx\ ,\ n\neq -m \quad \text{reduces } \cos^m x$$

$$\int\sin^n ax\cdot\cos^m axdx=-\frac{\sin^{n-1}ax\cdot\cos^{m+1}ax}{a(m+n)}+\frac{n-1}{m+n}\int\sin^{n-2}ax\cdot\cos^m axdx\ ,\ n\neq -m \quad \text{reduces } \sin^n ax$$

$$\int\sin^n ax\cdot\cos^m axdx=\frac{\sin^{n+1}ax\cdot\cos^{m-1}ax}{a(m+n)}+\frac{m-1}{m+n}\int\sin^n ax\cdot\cos^{m-2}axdx\ ,\ n\neq -m \quad \text{reduces } \cos^m ax$$

برهان تخفيض $\cos^m x$

$$\int \sin^n x \cdot \cos^m x dx = \int \sin^n x \cdot \cos^{m-1} x \cos x dx = \int \cos^{m-1} x \cdot \sin^n x \cdot \cos x dx$$

Let $u = \cos^{m-1} x \quad \Rightarrow \quad du = -(m-1)\cos^{m-2} x \sin x dx$ **&** $dv = \sin^n x \cdot \cos x dx \quad \Rightarrow \quad v = \frac{\sin^{n+1} x}{n+1}$

$$I = \int \sin^n x \cdot \cos^m x dx = \int u dv = uv - \int v du = (\cos^{m-1} x)(\frac{\sin^{n+1} x}{n+1}) + \int \frac{\sin^{n+1} x}{n+1}(m-1)\cos^{m-2} x \sin x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^{n+2} x \cdot \cos^{m-2} x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cdot \sin^2 x \cos^{m-2} x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x(1-\cos^2 x)\cos^{m-2} x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cos^{m-2} x dx - \frac{m-1}{n+1}\int \sin^n x \cos^m x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cos^{m-2} x dx - \frac{m-1}{n+1} I$$

$$I + \frac{m-1}{n+1} I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cos^{m-2} x dx$$

$$\frac{n+1+m-1}{n+1} I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cos^{m-2} x dx$$

$$\frac{n+m}{n+1} I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} + \frac{m-1}{n+1}\int \sin^n x \cos^{m-2} x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+1} \cdot \frac{n+1}{n+m} + \frac{m-1}{n+1} \cdot \frac{n+1}{n+m}\int \sin^n x \cos^{m-2} x dx$$

$$I = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{n+m} + \frac{m-1}{n+m}\int \sin^n x \cos^{m-2} x dx$$

بنفس الطريقة يمكننا إثبات الصيغ الأخرى للتخفيض، من خلال تطبيق هذه الصيغة بشكل متكرر ، يمكننا في النهاية ان نكامل بقوة منخفضة بما يكفي لحلها مباشرة. المثال التالي يوضح هذا الإجراء.

Example (59): Evaluate $\int (\sin x)^3 (\cos x)^2 dx$

- بما ان اس $\sin x$ عدد فردي فالسؤال من الطريقة الرابعة عشر الاحتمال الثاني لذلك نجزأ دالة $\sin x$

$$\int (\sin x)^3 (\cos x)^2 dx = \int (\cos x)^2 (\sin x)^3 dx = \int (\cos x)^2 (\sin x)^2 (\sin x) dx$$

$$= \int (\cos^2 x)(1-\cos^2 x)(\sin x) dx \;= \int (\cos^2 x - \cos^4 x)(\sin x) dx$$

$$= \int \cos^2 x \sin x dx - \int \cos^4 x \sin x dx \quad = \frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$$

- يمكن حل هذا السؤال باستخدام القاعدة

$$\int \sin^n x \cdot \cos^m x dx = -\frac{\sin^{n-1} x \cdot \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \sin^{n-2} x \cdot \cos^m x dx$$

$$\int (\sin x)^3 (\cos x)^2 dx = -\frac{\sin^2 x \cdot \cos^3 x}{5} + \frac{2}{5} \int \sin x \cdot \cos^2 x dx = -\frac{\sin^2 x \cdot \cos^3 x}{5} - \frac{2}{5} \int \cos^2 x(-\sin x) dx$$

$$\int (\sin x)^3 (\cos x)^2 dx = -\frac{\sin^2 x \cdot \cos^3 x}{5} - \frac{2}{5} \cdot \frac{\cos^3 x}{3} + c$$

- وكذلك يمكن حل هذا السؤال باستخدام القاعدة

$$\int \sin^n x \cdot \cos^m x dx = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{m+n} + \frac{m-1}{m+n} \int \sin^n x \cdot \cos^{m-2} x dx$$

$$\int (\sin x)^3 (\cos x)^2 dx = \frac{\sin^4 x \cdot \cos x}{5} + \frac{1}{5} \int \sin^3 x dx$$

$$\int (\sin x)^n dx = \frac{-(\sin x)^{n-1} \cos x}{n} + \frac{n-1}{n} \int (\sin x)^{n-2} dx$$

$$\int (\sin x)^3 (\cos x)^2 dx = \frac{\sin^4 x \cdot \cos x}{5} + \frac{1}{5}\left(\frac{-(\sin x)^2 \cos x}{3} + \frac{2}{3} \int \sin x dx \right)$$

$$\int (\sin x)^3 (\cos x)^2 dx = \frac{\sin^4 x \cdot \cos x}{5} - \frac{1}{5}\left(\frac{(\sin x)^2 \cos x}{3} + \frac{2}{3} \cos x \right) + c$$

Example (60): Evaluate $\int (\sin x)^2 (\cos x)^2 dx$

- بما ان اســس كل من دالة $\sin x$ ودالة $\cos x$ اعداد زوجية فهذا يعني ان الســؤال من الطريقة الرابعة عشر الاحتمال الثالث لذلك فنستخدم المتطابقتين الاتيتين

$$(\cos x)^2 = \frac{1+\cos 2x}{2} \qquad \& \qquad (\sin x)^2 = \frac{1-\cos 2x}{2}$$

$$\int (\sin x)^2 (\cos x)^2\,dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx = \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\,dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x\,dx = \frac{x}{4} - \frac{1}{4}\int \frac{1+\cos 4x}{2}dx$$

$$= \frac{x}{4} - \frac{1}{4}\int \left[\frac{1}{2} + \frac{\cos 4x}{2}\right]dx = \frac{x}{4} - \frac{1}{8}\int dx - \frac{1}{8}\int [\cos 4x]\,dx$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{8}\frac{1}{4}\int [\cos 4x](4)\,dx = \frac{x}{8} - \frac{1}{32}\sin 4x + c$$

● يمكن حل هذا السؤال باستخدام القاعدة

$$\int \sin^n x \cdot \cos^m x\,dx = \frac{\sin^{n+1} x \cdot \cos^{m-1} x}{m+n} + \frac{m-1}{m+n}\int \sin^n x \cdot \cos^{m-2} x\,dx$$

$$\int (\sin x)^2 (\cos x)^2\,dx = \frac{\sin^3 x \cdot \cos x}{4} + \frac{1}{4}\int \sin^2 x\,dx = \frac{\sin^3 x \cdot \cos x}{4} + \frac{1}{4}\int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$$

$$\int (\sin x)^2 (\cos x)^2\,dx = \frac{\sin^3 x \cdot \cos x}{4} + \frac{1}{4}\cdot\frac{x}{2} - \frac{1}{8}\int \cos 2x\,dx = \frac{\sin^3 x \cdot \cos x}{4} + \frac{x}{8} - \frac{1}{16}\int \cos 2x(2)\,dx$$

$$\int (\sin x)^2 (\cos x)^2\,dx = \frac{\sin^3 x \cdot \cos x}{4} + \frac{x}{8} - \frac{1}{16}\sin 2x + c$$

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Exercise (6-1): Evaluate the trigonometric integrals Calculus Exe. (8.2) - page 448 – questions(1-65)

No.	Question	Answer
	Powers of Sines and Cosines Evaluate the integrals in Exercises 1-22.	
1	$\int \cos 2x\,dx$	$I = \frac{1}{2}\sin 2x + c$
2	$\int_0^{\pi} 3\sin \frac{x}{3}\,dx$	$I = \frac{9}{2}$
3	$\int \cos^3 x \sin x\,dx$	$I = \frac{-1}{4}\cos^4 x + c$
4	$\int \cos 2x \sin^4 2x\,dx$	$I = \frac{1}{10}\sin^5 2x + c$
5	$\int \sin^3 x\,dx$	$I = -\cos x + \frac{1}{3}\cos^3 x + c$
6	$\int \cos^3 4x\,dx$	$I = \frac{1}{4}\sin 4x - \frac{1}{12}\sin^3 4x + c$
7	$\int \sin^5 x\,dx$	$I = -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$

8	$\int_0^{\pi} \sin^5 \frac{x}{2} dx$	$I = \frac{16}{15}$
9	$\int \cos^3 x dx$	$I = \sin x - \frac{1}{3}\sin^3 x + c$
10	$\int_0^{\pi/6} 3\cos^5 3x dx$	$I = \frac{8}{15}$
11	$\int \cos^3 x \sin^3 x dx$	$I = \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$
12	$\int \cos^3 2x \sin^5 2x dx$	$I = \frac{1}{12}\sin^6 2x - \frac{1}{16}\sin^8 2x + c$
13	$\int \cos^2 x dx$	$I = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$
14	$\int_0^{\pi/2} \sin^2 x dx$	$I = \frac{\pi}{4}$
15	$\int_0^{\pi/2} \sin^7 y dy$	$I = \frac{16}{35}$
16	$\int 7\cos^7 t dt$	$I = 7\sin t - 7\sin^3 t + \frac{21}{5}\sin^5 t - \sin^7 t + c$
17	$\int_0^{\pi} 8\sin^4 x dx$	$I = 3\pi$
18	$\int 8\cos^4 2\pi x dx$	$I = 3x + \frac{1}{\pi}\sin 4\pi x + \frac{1}{8\pi}\sin 8\pi x + c$
19	$\int 16\cos^2 x \sin^2 x dx$	$I = 2x - 4\sin x \cos^3 x + 2\sin x \cos x + c$
20	$\int_0^{\pi} 8\sin^4 y \cos^2 y dy$	$I = \frac{\pi}{2}$
21	$\int 8\cos^3 2\theta \sin 2\theta dx$	$I = -\cos^4 2\theta + c$
22	$\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta d\theta$	$I = 0$
Integrating Square Roots Evaluate the integrals in Exercises 23-32.		
23	$\int_0^{\pi/2} \sqrt{\frac{1-\cos x}{2}} dx$	$I = 4$
24	$\int_0^{\pi} \sqrt{1-\cos 2x} dx$	$I = 2\sqrt{2}$
25	$\int_0^{\pi} \sqrt{1-\sin^2 t} dt$	$I = 2$

26	$\int_{0}^{\pi} \sqrt{1-\cos^2\theta}\, d\theta$	$I=2$
27	$\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{\sqrt{1-\cos x}}\, dx$	$I=\sqrt{\frac{3}{2}}-\frac{2}{3}$
28	$\int_{0}^{\pi/6} \sqrt{1+\sin x}\, dx$ hint multiply by $\sqrt{\frac{1-\sin x}{1-\sin x}}$	$I=2-\sqrt{2}$
29	$\int_{5\pi/6}^{\pi} \frac{\cos^4 x}{\sqrt{1-\sin x}}\, dx$	$I=\frac{4}{5}(\frac{3}{2})^{5/2}-\frac{2}{7}(\frac{3}{2})^{7/2}-\frac{18}{35}$
30	$\int_{\pi/2}^{3\pi/4} \sqrt{1-\sin 2x}\, dx$	$I=\frac{\sqrt{2}-1}{\sqrt{2}}$
31	$\int_{0}^{\pi/2} \theta\sqrt{1-\cos 2\theta}\, d\theta$	$I=\sqrt{2}$
32	$\int_{-\pi}^{\pi} (1-\cos^2 t)^{3/2}\, dt$	$I=\frac{8}{3}$
Powers of Tangents and Secants Evaluate the integrals in Exercises 33-50.		
33	$\int \sec^2 x \tan x\, dx$	$I=\frac{1}{2}\tan^2 x+c$
34	$\int \sec x \tan^2 x\, dx$	$I=\frac{1}{2}\sec x\tan x-\frac{1}{2}\ln\left\|\sec x+\tan x\right\|+c$
35	$\int \sec^3 x \tan x\, dx$	$I=\frac{1}{3}\sec^3 x+c$
36	$\int \sec^3 x \tan^3 x\, dx$	$I=\frac{1}{5}\sec^5 x-\frac{1}{3}\sec^3 x+c$
37	$\int \sec^2 x \tan^2 x\, dx$	$I=\frac{1}{3}\tan^3 x+c$
38	$\int \sec^4 x \tan^2 x\, dx$	$I=\frac{1}{5}\tan^5 x+\frac{1}{3}\tan^3 x+c$
39	$\int_{-\pi/3}^{0} 2\sec^3 x\, dx$	$I=2\sqrt{3}-\ln(2-\sqrt{3})$
40	$\int e^x \sec^3 e^x\, dx$	$I=\frac{1}{2}[\sec(e^x)\tan(e^x)+\ln\left\|\sec(e^x)+\tan(e^x)\right\|]+c$
41	$\int \sec^4\theta\, d\theta$	$I=\frac{1}{3}\tan\theta\sec^2\theta+\frac{2}{3}\tan\theta+c$
42	$\int 3\sec^4 3x\, dx$	$I=\tan(3x)+\frac{2}{3}\tan^2(3x)+c$
43	$\int_{\pi/4}^{\pi/2} \csc^4\theta\, d\theta$	$I=\frac{4}{3}$

44	$\int \sec^6 x dx$	$I=\frac{1}{5}\tan^5(x)+\frac{2}{3}\tan^2(x)+\tan(x)+c$
45	$\int 4\tan^3 x dx$	$I=2\tan^2(x)-2\ln(1+\tan^2(x))+c$
46	$\int_{-\pi/4}^{\pi/4} 6\tan^4 x dx$	$I=3\pi-8$
47	$\int \tan^5 x dx$	$I=\frac{1}{4}\tan^4 x-\frac{1}{2}\tan^2 x+\ln\lvert\sec x\rvert+c$
48	$\int \cot^6 2x dx$	$I=\frac{-1}{10}\cot^5(2x)+\frac{1}{6}\cot^3(x)-\frac{1}{2}\cot(2x)+c$
49	$\int_{\pi/6}^{\pi/3} \cot^3 x dx$	$I=\frac{4}{3}-\ln\sqrt{3}$
50	$\int 8\cot^4 t dt$	$I=\frac{-8}{3}\cot^3(t)+8\cot(t)+8t+c$
Products of Sines and Cosines Evaluate the integrals in Exercises 51-56.		
51	$\int \sin 3x \cos 2x dx$	$I=-\frac{1}{2}\cos x-\frac{1}{10}\cos 5x+c$
52	$\int \sin 2x \cos 3x dx$	$I=\frac{1}{2}\cos x-\frac{1}{10}\cos 5x+c$
53	$\int_{-\pi}^{\pi} \sin 3x \sin 3x dx$	$I=\pi$
54	$\int_{0}^{\pi/2} \sin x \cos x dx$	$I=\frac{1}{2}$
55	$\int \cos 3x \cos 4x dx$	$I=\frac{1}{2}\sin x+\frac{1}{14}\sin 7x+c$
56	$\int_{-\pi/2}^{\pi/2} \cos x \cos 7x dx$	$I=0$
Exercises 57-62 require the use of various trigonometric identities before you evaluate the integrals.		
57	$\int \sin^2\theta \cos 3\theta d\theta$	$I=\frac{1}{6}\sin 3\theta-\frac{1}{4}\sin\theta-\frac{1}{20}\sin 5\theta+c$
58	$\int \cos^2 2\theta \sin\theta d\theta$	$I=-\frac{4}{5}\cos^5\theta+\frac{4}{3}\cos^3\theta-\cos\theta+c$
59	$\int \sin^3\theta \cos 2\theta d\theta$	$I=-\frac{2}{5}\cos^5\theta+c$
60	$\int \sin^3\theta \cos 2\theta d\theta$	$I=\frac{2}{5}\cos^5\theta-\cos^3\theta+\cos\theta+c$
61	$\int \sin\theta \cos\theta \cos 3\theta d\theta$	$I=\frac{1}{4}\cos\theta-\frac{1}{20}\cos 5\theta+c$
62	$\int \sin\theta \sin 2\theta \sin 3\theta d\theta$	$I=-\frac{1}{8}\cos 2\theta-\frac{1}{16}\cos 4\theta+\frac{1}{24}\cos 6\theta+c$

Assorted Integrations

Use any method to evaluate the integrals in Exercises 63-68.

63	$\int \frac{\sec^3 x}{\tan x} dx$	$I = \sec x - \ln\lvert\csc x + \cot x\rvert + c$
64	$\int \frac{\sin^3 x}{\cos^4 x} dx$	$I = \frac{1}{3}\sec^3 x - \sec x + c$
65	$\int \frac{\tan^2 x}{\csc x} dx$	$I = \sec x + \cos x + c$
66	$\int \frac{\cot x}{\cos^2 x} dx$	$I = -\ln\lvert\csc 2x + \cot 2x\rvert + c$
67	$\int x\sin^2 x dx$	$I = \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$
68	$\int x\cos^3 x dx$	$I = x\sin x - \frac{1}{3}x\sin^3 x + \frac{2}{3}\cos x + \frac{1}{9}\cos^3 x + c$

Exercise (6-2): Evaluate the trigonometric integrals Exe. (1-12) page- 389 – questions(1-20) blue book

No.	Question	Answer
1	$\int x^3 \cos x dx$	$I = x^3 \sin x + 3x^2 \cos x - 6x\sin x - 6\cos x + c$
2	$\int x\cosh x dx$	$I = x\sinh x - \cosh x + c$
3	$\int x^2 \ln x dx$	$I = \frac{x^3}{3}\ln x - \frac{x^3}{9} + c$
4	$\int \frac{x dx}{\sin^2 x}$	$I = -x\cot x + \ln\lvert\sin x\rvert + c$
5	$\int x^3 \ln x dx$	$I = \frac{x^4}{4}\ln x - \frac{x^4}{16} + c$
6	$\int e^{ax}\sin bx dx$	$I = \frac{b^2}{a^2+b^2}\left(\frac{-1}{b}e^{ax}\cos bx + \frac{a}{b^2}e^{ax}\sin bx\right) + c$
7	$\int x\sin 3x dx$	$I = \frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$
8	$\int \sin(\ln x) dx$	$I = \frac{-x\cos(\ln x) + x\sin(\ln x)}{2} + c$
9	$\int x^3 e^x dx$	$I = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$
10	$\int \ln(a^2 + x^2) dx$	$I = x\ln(a^2 + x^2) - 2x + 2a\tan^{-1}\frac{x}{a} + c$
11	$\int x^2 \cos(2x) dx$	$I = \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + c$
12	$\int x\sqrt{1+x} dx$	$I = \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + c$
13	$\int x^3\sqrt{x-1} dx$	$I = \frac{2}{9}(x-1)^{9/2} + \frac{6}{7}(x-1)^{7/2} + \frac{6}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + c$

14	$\int \sin^2 x dx$	$I = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$
15	$\int \sin x \sinh x dx$	$I = \frac{\sin x \cosh x - \cos x \sinh x}{2} + c$
16	$\int \sec^2 x dx$	$I = \tan x + c$
17	$\int \ln(x^2) dx$	$I = x\ln(x^2) - 2x + c$
18	$\int x \tan^{-1} x dx$	$I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1} x + c\right)$
19	$\int \frac{\ln x}{x^3} dx$	$I = \frac{-1}{2x^2}\ln x - \frac{1}{4x^2} + c$
20	$\int \frac{xe^x}{(1+x)^2} dx$	$I = xe^x\left(\frac{-1}{1+x}\right) + e^x + c$

Exercise (6-3): Evaluate the trigonometric integrals Exe. (33) page (282) - book al-samarrai

No.	Question	Answer
1	$\int \sin 3x \sin 2x dx$	$I = \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + c$
2	$\int \sin 3x \cos 5x dx$	$I = \frac{1}{4}\cos(-2x) - \frac{1}{16}\cos(8x) + c$
3	$\int \cos 4x \cos 2x dx$	$I = \frac{1}{4}\sin 2x + \frac{1}{12}\sin 6x + c$
4	$\int \cos 3x \sin(-x) dx$	$I = \frac{-1}{8}\cos(-4x) + \frac{1}{4}\cos 2x + c$
5	$\int \cos^3 3x dx$	$I = \frac{(\cos 3x)^2 \sin 3x}{9} + \frac{2}{9}\sin 3x + c$
6	$\int \cos^4 2x \sin^5 2x dx$	$I = \frac{1}{10}\cos^5 2x - \frac{1}{7}\cos^7 2x + \frac{1}{18}\cos^9 2x + c$
7	$\int \sqrt{\sin x}\cos^3 x dx$	$I = \frac{2}{7}(\sin^{3/2} x \cdot \cos^2 x) + \frac{8}{21}(\sin x)^{3/2} + c$
8	$\int \sin^3 x dx$	$I = \frac{-(\sin x)^2 \cos x}{3} - \frac{2}{3}(\cos x) + c$
9	$\int \sin^3 x (\cos x)^{3/2} dx$	$I = \frac{-2}{5}(\cos x)^{5/2} + \frac{2}{9}(\cos x)^{9/2} + c$
10	$\int \cos^2 3x dx$	$I = \frac{\cos 3x \cdot \sin 3x}{6} + \frac{1}{2}x + c$
11	$\int \cos^4 x \sin^5 x dx$	$I = -\frac{\sin^4 x \cdot \cos^5 x}{9} - \frac{4\sin^2 x \cdot \cos^5 x}{63} + \frac{8}{63}\frac{(\cos x)^5}{5} + c$
12	$\int \cos^2 x \sin^4 x dx$	$I = \frac{\sin^5 x \cdot \cos x}{6} + \frac{1}{6}\left(\frac{-(\sin x)^3 \cos x}{4} + \frac{-3\sin x \cdot \cos x}{8} + \frac{3}{8}x\right) + c$
13	$\int \sin^4 x dx$	$I = \frac{-(\sin x)^3 \cos x}{4} + \frac{-3\sin x \cdot \cos x}{8} + \frac{3}{8}x + c$

14	$\int \cos^6 3x dx$	$I=\frac{(\cos 3x)^5 \sin 3x}{18}+\frac{5(\cos 3x)^3 \sin 3x}{72}+\frac{5\cos 3x \cdot \sin 3x}{48}+\frac{15}{48}x+c$
15	$\int \tan^4 x dx$	$I=\frac{(\tan x)^3}{3}-\tan x+x+c$
16	$\int \tan^3 3x \sec^4 3x dx$	$I=\frac{\tan^4 3x}{12}+\frac{\tan^6 3x}{18}+c$
17	$\int \cot^3 2x dx$	$I=\frac{-(\cot x)^2}{4}-\frac{1}{2}\ln\lvert\sin 2x\rvert+c$
18	$\int \cot 3x \csc^4 3x dx$	$I=-\frac{\cot^2 3x}{6}-\frac{\cot^4 3x}{12}+c$
19	$\int \tan^2 x \sec^3 x dx$	$I=\frac{\sec^3 x \tan x}{4}-\frac{\sec x \tan x}{8}-\frac{1}{8}\ln\lvert\sec x+\tan x\rvert+c$
20	$\int \sqrt{\tan x}\sec^4 x dx$	$I=\frac{\tan^{3/2}}{3/2}+\frac{\tan^{7/2}}{7/2}+c$
21	$\int \frac{dx}{\tan x \sec x}$	$I=-\ln\lvert\csc x+\cot x\rvert+\cos x+c$
22	$\int \cos^{3/2} x \sin^5 x dx$	$I=-\frac{\cos^{5/2} x}{5/2}+2\frac{\cos^{9/2} x}{9/2}-13\frac{\cos^{13/2} x}{9/2}+c$
23	$\int \sec^4 x dx$	$I=\int (\sec x)^4 dx=\frac{(\sec x)^2(\tan x)}{3}+\frac{2}{3}\tan x+c$

(24) Prove **(a)** $\int_{-x}^{x} \sin mx \sin nx dx=0$ for any $m,n\in I$ and $m\neq n$

(b) $\int_{-x}^{x} \sin mx s \cos nx dx=0$ for any $m,n\in I$ and $m\neq n$

(25) Find the area between the curve $y=\frac{\cos^2 x}{\sqrt{\sin x}}$ and x-axis and $x=1/2$, $x=1/6$

(26) Find the area between the curve $y=\cot^2 2x \csc^2 2x$ and x-axis and $x=1/3$, $x=1/6$

Exercise (6-4): Evaluate the trigonometric integrals Exe. (4-12) page (405) blue book

No.	Question	Answer
1	$\int \frac{\cos^3 x}{\sqrt{\sin x}} dx$	$I=\frac{\sin^{1/2} x}{1/2}-\frac{\sin^{5/2} x}{5/2}+c$
2	$\int \sin 7x \cos 9x dx$	$I=\frac{1}{4}\cos(-2x)-\frac{1}{32}\cos(16x)+c$
3	$\int \sin\theta \cos^2\theta d\theta$	$I=\frac{\cos^3\theta}{3}+c$
4	$\int \sin^2 3x \cos 3x dx$	$I=\frac{\sin^3 3x}{9}+c$
5	$\int \sin x\sqrt{1+\cos x} dx$	$I=\frac{(1+\cos x)^{3/2}}{3/2}+c$

6	$\int \frac{\cos^3 x}{\sin^2 x}dx$	$I=\frac{-1}{\sin x}-\sin x+c=-\csc x-\sin x+c$
7	$\int \frac{\sin x}{2-\cos x}dx$	$I=\ln\lvert 2-\cos x+c\rvert$
8	$\int \tan^3 x \sec x dx$	$I=\frac{\sec^3 x}{3}-\sec x+c$
9	$\int \frac{\sec^2 2x}{1+\tan 2x}dx$	$I=\frac{1}{2}\ln\lvert 1+\tan 2x+c\rvert$
10	$\int \tan^3 2x dx$	$I=\frac{\tan^2 2x}{4}-\frac{1}{2}\ln\lvert\cos 2x\rvert+c$
11	$\int_0^{\pi/2} \frac{\cos x dx}{\sqrt{1+\cos x}}$	$I=2-\sqrt{2}(\ln\lvert\sqrt{2}+1\rvert$
12	$\int \tan^7 x \sec^4 x dx$	$I=\frac{\tan^8 x}{8}+\frac{\tan^{10} x}{10}+c$
13	$\int \sin^6 x dx$	$I=\frac{-(\sin x)^5\cos x}{6}-\frac{5(\sin x)^3\cos x}{24}-\frac{15\sin x\cos x}{48}+\frac{15}{48}x+c$
14	$\int \frac{\cos 2x dx}{\sin^4 x}dx$	$I=-\frac{\csc^3 x}{3}+\cot x+c$
15	$\int \sec^6 x dx$	$I=\frac{\sec^3 x\tan x}{5}+\frac{4\sec^2 x\tan x}{15}+\frac{8}{15}\tan x+c$