جامعة تكريت

كلية التربية للبنات

قسم الرياضيات

المرحلة : الرابعة

المادة : الاحصاء الرياضي

Critical Region

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Critical Region

Def.: The rang of values of the test statistic (sample point) that according to give test, call for rejecting the hypothesis being tested is called the (critical region) of the test denoted by C. There for C is the subset of the sample space, which in accordance with a prescribed test, leads to the rejection of the Null hypothesis $H_{0.}$

$\therefore \ \alpha = \int f_{\circ}\,(x)\ d(x)$

تكامل الدالة تحت ظروف فرضية العدم في المنطقة الحرجة = α

$\beta = \int f_{1\ (x)d(x)}$

تعني تكامل الدالة في منطقة القبول تحت الفرضية البديلة = β

1- $\beta = \int f_{1(x)d(x)}$

يسمى قوة الاختبار *p .o. T*

Thus α can be mode zero by choosing C $\neq$ Ø

that is a dopting a rule of never rejecting $H_{0.}$

But this implies $c^{c}\ \Omega\epsilon$, β =1.

similarly , when $C\ \ \Omega\epsilon$, then $\beta = 0$, and $\alpha = 1$

therefore to make α just close to zero it will be found that this tends to make β large and conversely.

Def . The power function

Of a Hypothesis $H_0 : \ \theta = \theta_0$ against an alternative hypothesis is that function denoted by K(θ) , which yields the probability of rejecting H_0 , given θ is true.

K(θ) = P{ reject H_0 : $\theta = \theta_0$, given θ is true }, therefor the value of the power function at parameter point is called the power of the test at that point.

In testing a hypothesis, the power function K (θ) will play the same rule of M.S.e (in the estimation) , to finding a good test or in comparing true test.

An ideal power function K(θ) will be K(θ_0) and K(θ_1) =1 because we don't want to reject H_0 , when H_0 is true and we 0 . want to reject H_0 when H_0 is false.

Remark : The probability that H_0 : $\theta = \theta_0$

is accepted given θ is true is called the Operating characteristic function (O.C) of the test.

i.e O.C (θ) = p{ accept H_0 : $\theta = \theta_0$, given is true }

Example (1) : Let x have the p.d.f

$$p(x,\theta) = \theta^x(1-\theta)^{1-x} \qquad x=0,1 \quad , 0 < \theta < 1$$

the test the simple hypothesis H_0 : $\theta = \frac{1}{4}$

against the alterative composite Hypothesis H_1 : $\theta < \frac{1}{4}$,

suppose that the critical region is :

C = { (x_1, x_2, ….., x_{10}) ; $\sum_{i=1}^{10} x_i \le 1$ }

find a- The power function k(θ) b- $\propto$ c- The power of this test at $\theta = \frac{1}{16}$ d- 0.c e- β $at\ \theta = \frac{1}{16}$

s0l. : since $x \sim B(1, \theta)$

Let y = $\sum_{i=1}^{10} x_i \rightarrow y \sim B(10, \theta)$

f (y) = $C_y^{10} \quad \theta^y (1-\theta)^{10-y}; \quad y = 0,1,2, \dots\dots, 10$

a – $\quad$ k(θ) = p{ y $\leq 1 / \theta$}

=p (y=0) + p (y =1)

= $C_0^{10}\ \theta^0\ (1-\theta)^{10}\ +\ \ C_1^{10}\ \theta^1\ (1-\theta)^9$

= $(1-\theta)^9 [\,1-\theta+\theta^{10}]$

b- $\quad$ k(θ_0 *)* = $C_0^{10} (\frac{1}{4})^0\ (\frac{3}{4})^{10}$ + $C_1^{10} (\frac{1}{4})^1\ (\frac{3}{4})^9$

= 13/4 $(\frac{3}{4})^9$

Power function of the test $(\theta = \frac{1}{16}\ )$

$$= \ C_0^{10}\ \left(\frac{1}{16}\right)^0 (\frac{15}{16})^{10} + \ C_1^{10} \frac{1}{16}) (\frac{15}{16})^9$$

K(θ = 1/16) = 25/16$(\frac{15}{16})^9$ اكتب المعادلة هنا.

O.C(θ) = 1- K(θ)

= 1- [$C_0^{10} \theta^0 (1-\theta)^{10} + C_1^{10} \theta^1 (1-\theta)^9$

$$\beta = 1 - 25/16(\frac{15}{16})^9$$

Example(2) : Let $x_1 , x_2 , \dots , x_n$ be distributed with N(θ ,1)

the test the simple hypothesis $H_0 : \theta = 0$, against $H_1 : \theta = 1$

The critical region is C = { (x_1, x_2, ….., x_{10}) ;$\bar{X} \geq k$}

Find n and k such that $\alpha\ = \beta = 0.01$

sol.

$x_i \sim \ N(\theta, 1)$

$\therefore \bar{X} \sim N(\theta\, , \frac{1}{n})$

$\bar{X} | H_0 \sim N(0\, , \frac{1}{n})$

$\bar{X} | H_1 \sim N(1\, , \frac{1}{n})$

بما أن قيمة $\alpha = 0.01$ تحت ظروف H_0

$$0.01 = p(z \geq \frac{k-0}{1/\sqrt{n}})$$

من جداول التوزيع الطبيعي

$$0.99 = p(z \leq \sqrt{n}k)$$

نجد أن :

$$\sqrt{n}k = 2.33 \ldots (1)$$

also $\beta = p(\bar{X} \leq k;\ \theta = 1)$

$$0.01 = p(z < \frac{k-1}{1/\sqrt{n}})$$

$$0.01 = p(z \leq \sqrt{n}(k-1))$$

من جداول التوزيع الطبيعي نجد أن :

$$\sqrt{n}(k-1) = -2.33 \ldots (2)$$

by solving 1 and 2 we find that

n= 22 , k =0.5

Example (3):

Let x_1, x_2 be a random sample for size 2from Exponential with parameter θ , to test the simple hypothesis $H_0 : \theta = 2$, against $H_1 : \theta = 4$; the critical region C = { (x_1, x_2) ; $x_1 + x_2 \geq 9.5$ }

find the probability of type I and type Ⅱ , as well as the power of the test.

sol.

$as \quad x_i \sim exp.(\theta) \quad for\ i = 1,2$

then $Y = x_1 + x_2 \sim \Gamma(2, \theta)$

$$g(y,\theta) = \frac{1}{\theta^2}\ ye^{-\frac{y}{\theta}} \quad , 0 < y < \infty$$

$$\alpha = p\{c\} = p\{y \geq 9.5;\ H_0\} = \int_{9.5}^{\infty} g(y;2)dy = \int_{9.5}^{\infty} \frac{1}{4}\ ye^{-\frac{y}{2}}dy = 0.05$$

$$\beta = p\{y < 9.5; H_1\} = \int_0^{9.5} g(y;4)dy = \int_0^{9.5} \frac{1}{16}\ ye^{-\frac{y}{4}}dy = 0.69$$

p.o.t. = 1-- β = 1—0.69 = 0.31