جامعة تكريت

كلية التربية للبنات

قسم الرياضيات

المرحلة : الثالثة

المادة : الاحصاء والاحتمالية

The Random Variables

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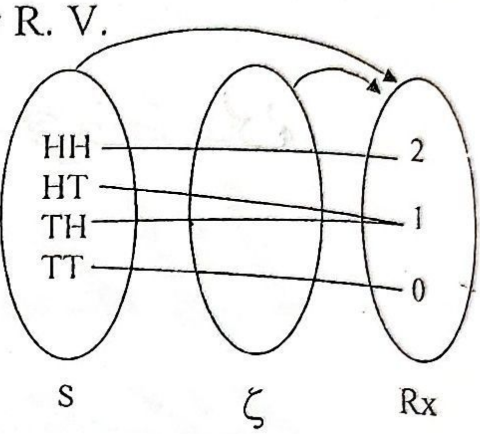
Def.: A random variable X is a function that mapps all elements $s \in S$ (all event in ζ) to a read numbers (Rx) denoted by R. V.



Ex 1.: Toss a coin twice.

Let X = number of H show that X is a R. V.

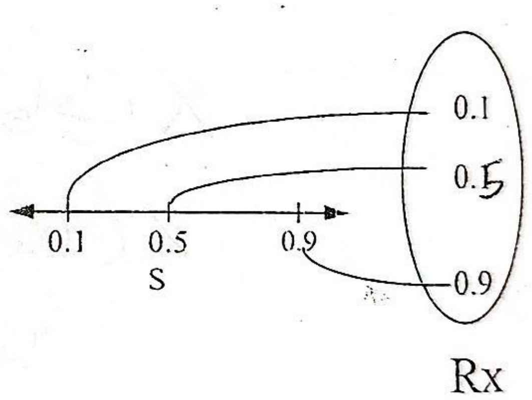
S = {HH, HT, TH, TT}

x(HH) = 2, x(HT) = 1, x(TH) = 1, x(TT) = 0

$Rx = \{x; x = 0, 1, 2\}$ countable

Note: We shall use X to denoted of R. V.X and x to denote of value of R. V. X, $x \in X$: 0, 1, 2 (in ex. "1").

Ex2.: Choose a point from interval (0, 1).



Let X be the chosen point, to show X is a R. V.

S consist all point in (0, 1)

∴ S has infinite number of points

The points in S are mapped to a real numbers.

then X is a R. V.

$Rx = \{x; 0 < x < 1\}$ uncountable.

Def.: A random variable X is a say to be discret r. v. if Rx is countable. See ex. "1" above, denoted by d. r. v.

Def.: A random variable x is say to be continuous r. v. if Rx is uncountable denoted by c.r.v.

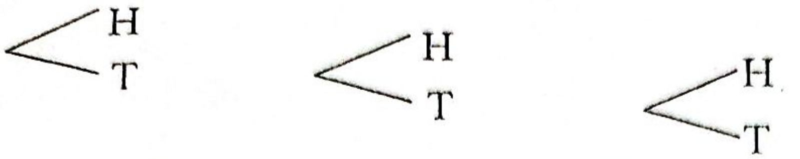
See ex, "2" above.

Ex.3: Toss a coin until first H appears.

Let x: number of tosses. show that x is d.r.v.

Sol.:

$x = 1$ $\qquad$ $x = 2$ $\qquad$ $x = 3$



$x \in X$: 1, 2, 3, 4, ...

$\therefore R_x = \{x; x \in N\}$ countable

$\therefore$ x is d.r.v.

Def.: Probability Mass function (P. M. f).

Let X be a d.r.v.

A function f is a p.m.f. of X if $f(x) = p(X = x)$, and satisfy the following conditions:-

1. $f(x) \geq 0, \forall x \in X$, 2. $\sum_{\forall x \in X} f(x) = 1$

Note: 1. condition (1) shows the graph of f(x) above of the x – axis.

2. Also, if $A \subset S$ then $P(x \in A) = \sum_{x \in A} f(x) = 1$

Ex.: Given $f(x) = \begin{cases} \frac{x}{10}, & \text{for } x = 0, 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$

Show that f(x) is a p.M.f.

cond. "1": T. p. $f(x) \geq 0 \ \forall x \in X$

$$f(0)=0,\ f(1)=\frac{1}{10},\ f(2)=\frac{2}{10},\ f(3)=\frac{3}{10},\ f(4)=\frac{4}{10}$$

$\therefore\ f(x) \geq 0 \ \forall x \in X$

$\therefore$ cond (1) satisfied.

cond "2": T.p $\sum_{x=0}^{4} f(x)=1$

$$\sum_{x=0}^{4} f(x)=0+\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\frac{4}{10}=\frac{10}{10}=1.$$

$\therefore$ f(x) is a p. M.f

* $p(x=1)=f(1)=\frac{1}{10}$

* $p(x=8)=f(8)=0$

* $p(x \geq 3)=p[(x=3) \cup (x=4)]=p(x=3)+p(x=4)$

$$=f(3)+f(4)=\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$$

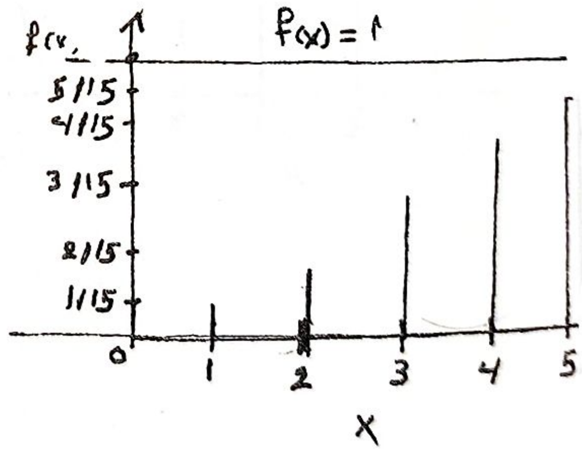
or by note 2 $p(x \geq 3)=\sum_{x=3}^{4} f(x)=f(3)+f(4)=\frac{7}{10}$

* $p(x \leq 2)=p[(x=2) \cup (x=1)]=p(x=2)+p(x=1)$

$$=f(2)+f(1)=\frac{2}{10}+\frac{1}{10}=\frac{3}{10}$$

Ex.: Given a p. m . f. $f(x)=\begin{cases} \frac{x}{k}, & \text{for } x=1,2,3,4,5 \\ 0, & \text{otherwise} \end{cases}$

find the value of k and sketch f(x).



Sol.: $\because$ f(x) is a p. m .f.

$\therefore$ by cond "2" we get $\sum_{x=1}^{5} f(x)=1$

$$f(1)+f(2)+f(3)+f(4)+f(5)=1$$

$$\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}+\frac{5}{k}=1 \Rightarrow \therefore k=15$$

$$\therefore f(x)=\begin{cases} \frac{x}{15}, & \text{for } x=1,2,3,4,5 \\ 0, & \text{otherwise} \end{cases}$$

Ex.: Toss a coin 3-times. Let x = number of H. find the p. M.f. of x and sketch it's graph.

Sol.: S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

∴ S has (8) elements

x = no. of H, $x \in X$; x= 0, 1, 2, 3

Rx = {x; x = 0, 1, 2, 3}, Rx is a countable

x is d. r. v.

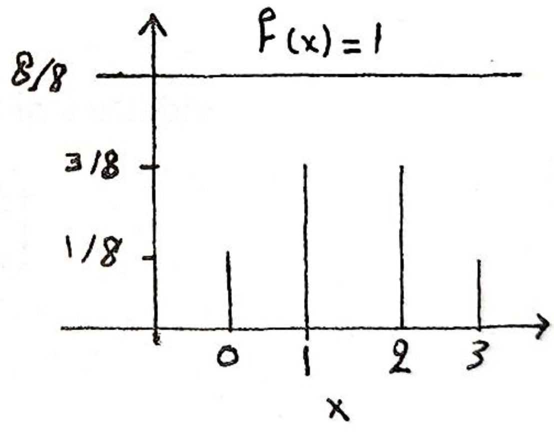
Event (X = x): to get xH, x = 0, 1, 2, 3 when toss a coin 3- times

$\binom{3}{x}$: number of samples in event (X = x) when toss a coin 3-times

$$f(x) = p(X = x) = \begin{cases} \dfrac{\binom{3}{x}}{8} & \text{for } x = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

x	$f(x) = \dfrac{\binom{3}{x}}{8}$
0	1/8
1	3/8
2	3/8
3	1/8

$$\sum_{x=0}^{3} f(x) = 1$$



Def.: "probability distribution"

A probability dist. of a r.v.X is a set of all ordered pair of x and f(x), $\forall x \in X$.

i. e.: pr. dist.of x = $\{(xi, f(xi)); \forall xi \in X\}$ of X.