جامعة تكريت
كلية التربية للبنات
قسم الرياضيات

محاضرة بعنوان

(التشاكل الحلقي)

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[redacted]

Definition :- Let f: (R , +, .) ⟶ (R\`, +\`, .\`) be a function . Then

(1) f is called **Monomorphism** iff f is one to one and homo .

(2) f is called **Epimorphism** iff f is onto and homo.

(3) f is called **Isomorphism** iff f is one to one , onto and homo .

(4) f is called **Endomorphism** iff f is homomorphism and R=R̀ .

(5) f is called **Automorphism** iff f is isomorphism and R=R̀ .

Definition: - **(Isomorphic Rings)** الحلقات المتماثلة

Two rings (R , +, .) and (R\`, +\` , .\` ) are said to be **isomorphic** if there exists f : R⟶ R\` such that f is an isomorphism , and is denoted by (R≅ R\`) .

Example(1):- Let f: (Z ,+ , .) ⟶ (Z , + , .) be a function , S.t f(n) = n. 1 ,∀ n ∈ Z . Show that : (f is ring homo , Epimorphism , Monomorphism , Isomorphism , Endomorphism and Automorphism)

Solution:- **First to show f is ring homo**

Let n ,m ∈ Z ⟹ f (n) = n. 1 and f (m) = m. 1 , thus

1- f(n + m) =(n + m) . 1

=(n. 1) +(m.1) = f (n) + f (m)

2- f(n. m) = (n. m). 1

$=(n . m).1^2$

= (n ,m) 1.1

= (n.1). (m.1)= f(n) . f(m)

∴ f is ring homomorphism ⋯⋯ **(i)**

< To show f is Epimorphism >? → f is ring homo; f is on to

by step(i), f is ring homo

< Now, to show f is on to>?

$\because f(Z) = \{ f(n), \forall n \in Z \}$

$= \{ n.1, \forall n \in Z \}$

$= \{ n, \forall n \in Z \} = Z$

$\Rightarrow$ **f is on to ⋯⋯ (ii)**

$\Rightarrow$ **f is Epimorphism .**

Now ,to show , f is Monomorphism → f is ring homo; f is on to one

by step(i), f is ring homo

< To show f is on to one >?

let $n, m \in Z$ such that $f(n) = f(m)$ **< T.P. n = m >?**

$\because f(n) = f(m)$

$\therefore n.1 = m.1$

$\Rightarrow n = m$,

$\Rightarrow$ f is one to one ⋯⋯ **(iii)**

$\Rightarrow$ **f is Monomorphism .**

< To show f is Isomorphism >?

by step (i), (ii) and (iii) we get :

f is ring homo, onto and one to one

$\Rightarrow$ f is Isomorphism ⋯⋯ (v)

< To show f is Endomorphism >?

Since, R= Z and Ŕ=Z

$\Rightarrow$ R= Ŕ ⋯⋯ (iv)

Then, we get by step(i)[f is ring homo] and by step (iv) [R= Ŕ]

$\Rightarrow$ f is Endomorphism.

< To show f is Automorphism >?

By step(v)[f is isomorphism] and step(iv) [R= Ŕ], then

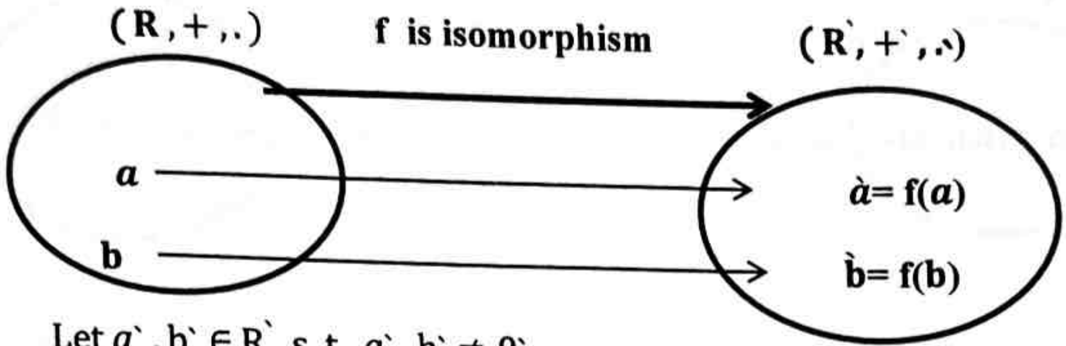
$\Rightarrow$ f is Automorphism.

Exc. (2):- Let f: $(\mathcal{R}, +, .) \longrightarrow (\mathcal{R}, +, .)$ be a function such that $f(a) = 0, \forall\, a \in \mathcal{R}$ Is f Endomorphism ? واجب

Theorem (3.9):- Every isomorphic image of a ring without zero divisors is a ring without zero divisors .

Proof: Let $f:(R,+,.) \longrightarrow (R',+',.')$ be an isomorphism function and $(R,+,.)$ is a ring without zero divisors

<T.P. $(R',+',.')$ is a ring without zero divisors>?



Let $a', b' \in R'$ s. t . $a', b' \neq 0'$

$\because$ f is 1-1 and on to $\Longrightarrow \exists!\, a, b \in R$ **s.t.** $\boxed{a' = f(a)}$ and $\boxed{b' = f(b)}$

Since, R without zero divisors

$\Rightarrow a \cdot b \neq 0$

$\Rightarrow f(a \cdot b) \neq f(0)$

$\Rightarrow f(a) \cdot' f(b) \neq 0'$ (since f is homo . by Th (3-1) $f(0)= 0'$)

$\Rightarrow a' \cdot' b' \neq 0'$

Therefore, $(R',+',.')$ is a ring without zero divisors . □

Theorem (3-10):- Every isomorphic image of field is field . واجب