جامعة تكريت – كلية التربية للبنات –قسم الرياضيات

المرحلة : الرابعة

المادة: التبولوجيا العامة

# عنوان المحاضرة : الخواص الوراثية في الفضاء التبولوجية

مدرس المادة : أ .د. رنا بهجت ياسين

الايميل الجامعي : Zain 2016@ tu.edu.iq

Separation axioms that can be moved and raised by adding some conditions to the maps.

**Theorem (39)**

If $((N,\sigma)$ is a $T_i$_space. Let $f:(M,\tau)\to(N,\sigma)$ be an injective $Con$ map, then $(M,\tau)$ is $T_i$ _space. Where i=0,1,2

**Proof.**

We prove that i=0

Let $u\neq v\in M$ then $f(u)\neq f\ (v)$ in $(N,\sigma)$(because $f$ is a injective). Since $(N,\sigma)$is a $T_0$_space, then $\exists\ H$ is $open$ set of $(N,\sigma)$such that $f(u)\in H$ and $f\ (v)\notin H$.

Since $f$ is a $Con$, therefore $f^{-1}(H)$ is a $open$ set in $(M,\tau)$ containing $u$ but not $v$, $u\in f^{-1}(H)$, $v\notin f^{-1}\ (H)$. Hence $(M,\tau)$ is a $T_0$_space.

in the some way prove i=1,2■

**Proposition(40)**

Let $f:(M,\tau)\to(N,\sigma) be$ surjective OM and if$(M,\tau)$ is a $T_i$_space, then $(N,\sigma)$is a $T_i$_space, where $i=0,1,2$.

**Proof.**

To prove that $i=1$

Assume that $\dot{u}\neq\dot{v}\in N$ .Since $f$ is a surjective, Then $\exists u\neq v\in M\ni\ \dot{u}=f(u)\ \&\ \dot{v}=f\ (v)$ and $f(u)\neq f\ (v)$.

We get $\exists\ H,D\ are\ two\ open$ sets of $(M,\tau)$ such that $u\in H\wedge v\notin H$ and $u\notin D\wedge v\in D$ (because $(M,\tau)$ is a $T_1$_space).

Hence $f(u)\in f(H), f\ (v)\in f\ (D)$,since $f$ is a OM, hence $f\ (u), f\ (v)$ are $open$ sets of $(N,\sigma)\ni\ \dot{u}\in f(H)$,but $\dot{v}\notin f(D)$and $\dot{v}\in f(D)$ but $\dot{u}\notin f\ (H)$. Then $(N,\sigma)$ is a $T_1$_space.

In the some way prove i=0,2. ■

**Theorem (41)**

Every compact subset of $T_2$_space is a closed.

**Proof.**

Let $(M, \tau)$ be $T_2$_space , $F$ be_compact subset of $M$ and let $p \in E = M - F, \forall\, x \in F$. Then $\exists$ disjoint open subsets $H_x, D_x$ containing $x$and $p$ respectively. $F = \bigcup_{x \in F}\{x\} \subseteq \bigcup_{x \in F} H_x$ ,there four $\{H_x\}$ $x \in F$ is open cover of $F$ , but $F$ is compact. Then $\exists\, x_1, \dots, x_n$, $F \subseteq \cup H_{x_1} \cup H_{x_2} \dots \cup H_{x_n} = H$ and $D = D_{x_1} \cap D_{x_2} \dots \cap D_{x_n}$ , $D$ is open such that $p \in D$ , $D \cap F = \emptyset$ if not.

Let $t \in D \cap F \to t \in D \wedge t \in F$. Then $t \in D_{xi}$ , $\forall i = 1,2, \dots, n$ and $t \in H_{xj}$ for some $j, 1 \leq j \leq n$.

We get $D \cap F = \emptyset \to D \subseteq E$ and $p$ is an interior point of $E$. Hence $F$ is a closed. ■

**Remarks (42)**

1. The set $E$ is compact subset of $T_2$space $(M, I(X))$ and $p \notin E$ , $p \in M$ then $\exists$ two disjoint open sets $H, D$ of $M \ni E \subseteq H \wedge p \in D$.
2. Let $f: (M, \tau) \to (N, \sigma)$be $Con$ map, if $A \subseteq M$ is compact relative to $(M, \tau)$ ,then $f\,(A)$ is compact.
3. The set closed subset of compact space is compact.

**Proposition (43)**

Let $(M, \tau)$ be compact space and $(N, \sigma)$be $T_2$_space, if $f: (M, \tau) \to (N, \sigma)$is bijective and $Con$, then $f$ is $Home$.

**Proof.**

To prove that $f$ is $Home$, it is enough to show that $f^{-1}$ is $Con$.

For this we must show that $f(F)$ is $closed$ set in $(N,\sigma)$, for any closed subset in $(M,\tau)$

Being a $closed$ subset of a compact set $(M,\tau)$ , $F$ is compact set. then $f(F)$ is compact subset of $T_2$_space $(N,\sigma)$ and $f(F)$ is $closed$ set, for any $F \subseteq M$ is closed set which implies $f(F) \subseteq N$ is $closed$ set, this $f^{-1}$ is $Con$.

Then $f$ is $Home$ . because [$f$ are bijective, $Con$and $f^{-1}$ is $Con$ ■

**<u>Theorem (44)</u>**

Let $f:(M,\tau) \to (N,\sigma)$be $Con$ and bijective map, if $(M,\tau)$ compact space and $(N,\sigma)$ is $T_2$_space. Then $M$ and$N$ are homeomorphic.

**Proof**.

Let $f(A)$ be $open$set in $(N,\sigma)$, since $(M,\tau)$ is compact, ∀ open cover there corresponds a finite sub cover and $M$ is $T_2$, for any $u \neq v \in M$, $\exists$ $two\ disjoint\ H\ and\ D$ are $open$ sets in $(N,\sigma)$ such that, $f(u) \in H, f\ (v) \in D$.
Then $open$ set $A$ in $(M,\tau)$ , $f\ (A)$ is $open$ set in $(N,\sigma)$ and $M - f(A)$ is $closed$ set in$(N,\sigma)$ .
Now to prove $f^{-1} = h: N \to M$ is $Con$ . Also, to prove $h^{-1}(A)$ is $open$( closed) set in $(N,\sigma)$ .
Since $A$ is open set and $M - A$ is closed set in $(M,\tau)$ , therefore $h^{-1}(M - A)$= $M - h^{-1}(A)$, we have $h^{-1}(N - A)$= $M - f(A)$--(i) [because $h^{-1} = f$ ]. From (i) for each closed set in $(M,\tau)$ we get $h^{-1}$ is closed set set in (($N,\sigma$)and $h$ is $Con$ map.

Therefore, $f$ is $Home$, then $M$ and $N$ are homeomorphic. ■