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عنوان المحاضرة : بديهيات الفصل في الفضاء التبولوجية

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Definition (28)

Topological space (M,τ) is T_0-axiom of separation briefly ($T_{0\_}$ $space$) for each a distinct points $v,u \in M$,if there exists a open set containing one of them but not the other.

Definitions (29)

1. Topological space $(M,\tau)$is said to satisfy the T_1 axiom of separation briefly ($T_{1\_}$ $space$) for each a distinct points $v,u \in M$,if there exists two open sets containing one of the two points but not the other.
2. Topological space $(M,\tau)$is said to satisfy the T_2 axiom of separation briefly ($T_{2\_}$ $space$) for each a distinct points $v,u \in M$,if there exists H,D are two disjoint $open$ sets such that $v \in H, u \in D$.

Theorem (30)

Every T_i_space is T_{i-1} _space, where $i = 1,2$.

Proof.

We prove that the theorem for $i = 1$

Let (M,τ) be a T_1_space if for v,u of M, there exist two disjoint $open$ sets H,D containing one of the two points, but not the other. Furthermore it there exist H is $open$ set, such that $v \in H$ and $u \notin H$. Then $(M,\tau)$is T_0_space.

We prove that the theorem for $i = 2$

Suppose that $(M,\tau)$is T_2_space, if for v,u of M, then there exists two disjoint $open$ sets containing one of the two points, but not the other. So there exists H,D

are open sets, such that $v \in H \; and \; u \notin H$ or $u \in D \; and \; v \notin D$. Then $(M, \tau)$is T_1_space. ■

Theorem (31)

The space (M, τ) is T_0_space if and only if for any distinct points v, u of M such that $cl\{v\} \neq cl\{u\}$.

Proof.

For $v, u \in M, v \neq u$ with $cl\{v\} \neq cl\{u\}$. Suppose that $w \in M$ such that $w \in cl\{v\}$, $w \notin cl\{u\}$. Therefore $v \notin cl\{u\}$. If $v \in cl\{u\}$ then $\{v\} \subseteq cl\{u\} \rightarrow cl\{v\} \subseteq cl\{u\}$. Thus $w \in cl\{v\} \wedge w \in cl\{u\}$ this is contradiction. Hence $M - cl\{u\}$ is $open$ set containing v, but not u. Then (M, τ) is T_0^{NP}_space.

Conversely,

Suppose that $v, u \in M$, $v \neq u$, since (M, τ) is a T_0_space and

$X \in O(M)$ such that $v \in X \wedge u \notin X$, therefore $cl\{u\} \subseteq M - X$. Hence $v \in M - X$ as $v \notin cl\{v\} \wedge u \in cl\{u\}$and $cl\{v\} \neq cl\{u\}$ ■

Proposition (32)

The surjective Con_{N_P}-image of T_0_space is T_0_space.

Proof.

Suppose that $f: (M, \tau) \rightarrow (N, \sigma)$ and $d, e \in M$, if f is onto then $\exists c , z$ such that $c = f^{-1}(d) \wedge z = f^{-1}(e)$. Since M is a T_0_space, then there exist open set containing one of the two points $c \; and \; z$ but not the other, for $c \in H \wedge z \notin H$ or S such that $z \in S \wedge c \notin S$, so

$f^{-1}(c) \in f(H) = H$ or $f^{-1}(z) \in f(S$)= S ,then $d \in H \wedge e \notin H$ or $e \in S \wedge d \notin S$. Then (M, τ) is a T_0_space.

Remark (33)

The following figure. Explains the relations between space T_i space , where i=0,1,2.

$$\mathrm{T}_2\_\textbf{space} \rightarrow \mathrm{T}_1\_\textbf{space} \rightarrow \mathrm{T}_0\_\textbf{space}$$

By adding some conditions to the function, we get the following theorems.

Theorem

Let $f:(M,\tau) \rightarrow (N,\sigma)$ be a bijective open map and $\mathbb{X}$ is T_i_ spaces, then $\mathbb{Y}$ is T_i_spaces, where i =0,1,2.

Proof. We prove the case i =2.

Let v_2, u_2 be two points in $\mathbb{Y}$ and $v_2 \neq u_2$. Since $\mathfrak{f}$ is bijective , then $\exists v_1, u_1 \in \mathbb{X}$ and $\mathfrak{f}(v_1) = v_2, \mathfrak{f}(u_1) = u_2$. But , $\mathbb{X}$ is T_2_ spaces, then $\exists$ two disjoint open sets $\mathbb{H}, \mathbb{D} \in \mathbb{X}$,whenerver $v_1 \in \mathbb{H}, u_1 \in \mathbb{D}$. Then $\mathfrak{f}(\mathbb{H}), \mathfrak{f}(\mathbb{D})$ are open sets in $\mathbb{Y}$, we get $v_2 \in \mathfrak{f}(\mathbb{H}), u_2 \in \mathfrak{f}(\mathbb{D})$ and $\mathfrak{f}(\mathbb{H}) \cap \mathfrak{f}(\mathbb{D}) = \emptyset$. Hence $\mathbb{Y}$ is T_2- space. ■

Theorem

If $f:(M,\tau)\rightarrow(N,\sigma)$ is injectivecont. map and $\mathbb{Y}$ is T_i_spaces, then $\mathbb{X}$ is T_i_spaces, where i=0,1,2.

Proof.

We prove the case i =2.

Suppose that v, u of $\mathbb{X}$ and $v \neq u$. Since $\mathfrak{f}$ is one to one, then $\mathfrak{f}(v) \neq \mathfrak{f}(u)$ in $\mathbb{Y}$. But $\mathbb{Y}$ is T_2_space, then $\exists$ two disjoint open sets $\mathbb{H}, \mathbb{D} \in \mathbb{Y}$, whenever,$\mathfrak{f}(v) \in \mathbb{H}, \mathfrak{f}(u) \in \mathbb{D}$.Then $\mathfrak{f}^{-1}(\mathbb{H}), \mathfrak{f}^{-1}(\mathbb{D})$ are open,we get $v \in \mathfrak{f}^{-1}(\mathbb{H})$, $u \in \mathfrak{f}^{-1}(\mathbb{D})$ and $\mathfrak{f}^{-1}(\mathbb{H}) \cap \mathfrak{f}^{-1}(\mathbb{D}) = \emptyset$.Then $\mathbb{X}$ is T_2_space. ■

<u>Theorem</u>

Let $f:(M,\tau)\rightarrow(N,\sigma)$ be injective irresolute map and $\mathbb{Y}$ is T_i_ spaces, then $\mathbb{X}$,is T_i_ spaces, where i=0,1,2.

Proof.

We prove the case i=0.

Let v, u in $\mathbb{X}$ and $v \neq u$ .Since $\mathfrak{f}$ is one to one , $\mathfrak{f}(v) \neq \mathfrak{f}(u)$ in $\mathbb{Y}$, $\mathbb{Y}$ is T_0 _ space, then $\exists$ an open set $\mathbb{H} \in \mathbb{Y}$, whenever $\mathfrak{f}(v) \in \mathbb{H}, \mathfrak{f}(u) \notin \mathbb{H}$.

Then $\mathfrak{f}^{-1}(\mathbb{H})$ is semi set (because $\mathfrak{f}$ is irresolute and every open is sem . set), we get $v \in \mathfrak{f}^{-1}(\mathbb{H})$, $u \notin \mathfrak{f}^{-1}(\mathbb{H})$.

Then $\mathbb{X}$ is T_0- space. ■