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مدرس المادة : أ .د. رنا بهجت ياسين

الايميل الجامعي : Zain 2016@ tu.edu.iq

In the following Propositions, we characterize continuous functions in terms of inverse image of $closuer$ and interior.

Proposition (5)

A map $f:(M,\tau)\rightarrow(N,\sigma)$ is Con iff $\mathfrak{f}(\mathrm{cl}(\mathbb{A}))\subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$ for each $\mathbb{A}\subseteq\mathcal{M}$.

Proof.

$\forall\ \mathbb{A}\subseteq\mathcal{M}$ and $\mathfrak{f}:\mathcal{M}\rightarrow$ N is a Con. map, so $\mathfrak{f}(\mathbb{A})\subseteq$ N.Then

cl $(\mathfrak{f}(\mathbb{A}))$ is$closed$ in N. we get $\mathfrak{f}^{-1}(\mathrm{cl}(\mathfrak{f}(\mathbb{A})))$

is $closed$ in $\mathcal{M}$.

Since $\mathfrak{f}(\mathbb{A})\subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))\rightarrow \mathbb{A}\subseteq \mathfrak{f}^{-1}(\mathrm{cl}\ \ (\mathfrak{f}(\mathbb{A})))$ and

cl $(\mathbb{A})\ \subseteq \mathrm{cl}(\mathfrak{f}^{-1}(\mathrm{cl}\ \ (\mathfrak{f}(\mathbb{A})))=\mathfrak{f}^{-1}(\mathrm{cl}(f(\mathbb{A}))$.

Then $\mathfrak{f}(\mathrm{cl}(\mathbb{A}))\subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$ for each $\mathbb{A}\subseteq\mathcal{M}$.

Conversely,

$\forall\ \mathbb{A}\subseteq\mathcal{M}$ which that, $\mathfrak{f}(\mathrm{cl}(\mathbb{A}))\subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$. If $\mathbb{D}$ is cl in N.

By hypothesis $\mathfrak{f}(\mathrm{cl}\left(\mathfrak{f}^{-1}(\mathbb{D})\right))\subseteq \mathrm{cl}(\mathfrak{f}\left(\mathfrak{f}^{-1}(\mathbb{D})\right))(\ \mathrm{cl}(\mathbb{D})=\mathbb{D})$

which implies that cl $\left(\mathfrak{f}^{-1}(\mathbb{D})\right)\subseteq\ \mathbb{D}$.

But always $\mathfrak{f}^{-1}(\mathbb{D})\subseteq$ cl $\left(\mathfrak{f}^{-1}(\mathbb{D})\right)$, so that cl $\left(\mathfrak{f}^{-1}(\mathbb{D})\right)=\mathfrak{f}^{-1}(\mathbb{D})$,

we get $\mathfrak{f}^{-1}(\mathbb{D})$ is $closed$ in N. Then $\mathfrak{f}$ is Con. map ■

In the following Propositions, we characterize Con. maps in terms of inverse image of CL and int.

Proposition (6)

A map $f:(M,\tau)\to(N,\sigma)$ is a con iff $\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{A})\big)\subseteq\mathfrak{f}^{-1}(\mathrm{cl}\ (\mathbb{A}))$, for each $\mathbb{A}\subseteq$ N.

Proof.

If $\mathfrak{f}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$ and $\mathbb{A}\subseteq\mathrm{cl}_{\mathcal{N}_{\mathcal{P}}}(\mathbb{A})\subseteq$ N, then $\mathfrak{f}^{-1}(\mathrm{cl}(\mathbb{A})$ is $closed$ in $\mathcal{M}$. So we get $\mathrm{cl}_{\mathcal{N}_{\mathcal{P}}}(\mathfrak{f}^{-1}\big(\mathrm{cl}(\mathbb{A})\big))=\mathfrak{f}^{-1}(\mathrm{cl}(\mathbb{A}))$, since $\mathbb{A}\subseteq\mathrm{cl}(\mathbb{A})$, $\mathfrak{f}^{-1}(\mathbb{A})\subseteq(\mathfrak{f}^{-1}\big(\mathrm{cl}(\mathbb{A})\big))$. Then $\mathrm{cl}\left(\mathfrak{f}^{-1}\big((\mathbb{A})\big)\right)\subseteq\mathrm{cl}\ \left(\mathfrak{f}^{-1}\big(\mathrm{cl}(\mathbb{A})\big)\right)=\mathfrak{f}^{-1}(\mathrm{cl}(\mathbb{A}))$. Hence $\mathrm{cl}(\mathfrak{f}^{-1}(\mathbb{A}))\subseteq\mathfrak{f}^{-1}(\mathrm{cl}(\mathbb{A}))$.

Conversely,

Suppose that cl $(\mathfrak{f}^{-1}(\mathbb{A}))\subseteq\mathfrak{f}^{-1}(\mathrm{cl}_{\mathcal{N}_{\mathcal{P}}}(\mathbb{A}))$ for each $\mathbb{A}\subseteq$ N and $\mathbb{A}$ is $closed$inN, such that $\mathbb{A}=\mathrm{cl}\ (\mathbb{A})$,

so $\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{A})\big)\subseteq\mathfrak{f}^{-1}\big(\mathrm{cl}(\mathbb{A})\big)=\mathfrak{f}^{-1}(\mathbb{A})\to\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{A})\big)\subseteq\mathfrak{f}^{-1}(\mathbb{A})$.

But $\mathfrak{f}^{-1}(\mathbb{A})\subseteq\mathrm{cl}\big(f^{-1}(\mathbb{A})\big)$, we get $\mathfrak{f}^{-1}(\mathbb{A})=\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{A})\big)$, that is $\mathfrak{f}^{-1}((\mathbb{A})$ is $closed$ in $\mathcal{M}$, for each $\mathbb{A}$ is $closed$ in N . Then $\mathfrak{f}$ is a Con. ■

Proposition (7)

A map $f:(M,\tau)\to(N,\sigma)$ is Con. iff $\mathfrak{f}^{-1}\big(\mathrm{int}\ (\mathbb{A})\big)\subseteq\mathrm{int}(\mathfrak{f}^{-1}(\mathbb{A}))$ for each $\mathbb{A}\subseteq$ N.

Proof.

If $\mathbb{A}\subseteq$ Nand $\mathfrak{f}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$, then int $(\mathbb{A})$is $closed$ in N, so we get $\mathfrak{f}^{-1}(\mathrm{int}\ (\mathbb{A}))$ is $closed$ in $\mathcal{M}$, that is $\mathfrak{f}^{-1}\big(\mathrm{int}\ (\mathbb{A})\big)=\mathrm{int}[\mathfrak{f}^{-1}(\mathrm{int}\ (\mathbb{A})]\subseteq\big(\mathfrak{f}^{-1}(\mathbb{A})\big)$ (because $\mathrm{int}\ (\mathbb{A})\subseteq\mathbb{A}$). Hence $\mathfrak{f}^{-1}(\mathrm{int}\ (\mathbb{A}))\subseteq\mathrm{int}(\mathfrak{f}^{-1}(\mathbb{A}))$.

Conversely,

Suppose $\mathfrak{f}^{-1}\big(\mathrm{int}\ (\mathbb{A})\big)\subseteq\mathrm{int}(\mathfrak{f}^{-1}(\mathbb{A}))$ for each $\mathbb{A}\subseteq$ N and $\mathbb{A}$ is $closed$ in N such that $\mathbb{A}=\mathrm{int}\ (\mathbb{A})$ therefore, $\mathfrak{f}^{-1}(\mathbb{A})\subseteq\mathrm{int}\ (f^{-1}(\mathbb{A}))$.

But int $(\mathfrak{f}^{-1}(\mathbb{A}))\subseteq\mathfrak{f}^{-1}(\mathbb{A})$, so $\mathfrak{f}^{-1}(\mathbb{A})=\mathrm{int}\ (\mathfrak{f}^{-1}(\mathbb{A}))$.

Thus $\mathfrak{f}^{-1}(\mathbb{A})$ is $closed$in $\mathcal{M}$, for each $\mathbb{A}$ is $closed$in $\mathcal{M}$. Then $\mathfrak{f}$ is a Con■

In the following Proposition, we characterize Conmaps in terms of basis elements.

Proposition (8)

Let $(M,\tau),(N,\sigma)$ and $\left(\bar{M},I(\bar{X})\right)$ are three topological spaces. If $f:(M,\tau)\rightarrow(N,\sigma)$ and g: $(N,\sigma)\rightarrow\left(\bar{M},I(\bar{X})\right)$ are Con functions, then g$of:(M,\tau)\rightarrow\left(\bar{M},I(\bar{X})\right)$ is Con.

Proof.

Suppose that H is open set in $\left(\bar{M},I(\bar{X})\right)$. Since g is Con, whenever $g^{-1}(H)$ is open set in (N,σ) . Now, $(gof)^{-1}H=f^{-1}(g^{-1}(H))$. So $f^{-1}(H)$is $open$ set in $\left(M,I(X)\right)$because $g^{-1}(H)$ is $open$ set in (N,σ) Since f is con. Function, then $gof:(M,\tau)\rightarrow\left(\bar{M},I(\bar{X})\right)$ is Con ■

The study present the definition and characterization of continuous map in terms of inverse image of closed.

Theorem(8)

Let W be $closed$ set in (N,σ) and let $f:(M,\tau)\rightarrow(N,\sigma)$ is Con map if and only if $f^{-1}(W)$ is $closed$ set in M,for each closed set W in (N,σ) .

Proof.

Suppose that $f:(M,\tau)\rightarrow(N,\sigma)$ is Con and W is closed set in (N,σ) , we get W^c is $open$ set in (N,σ) .

Since $f^{-1}(W^c)=f^{-1}(W)^{\,c}$ is $open\ set$ in (M,τ), hence $f^{-1}(W)$ is $closed$ set in (M,τ)whenever W is closed set in(N,σ) .

Conversely,

For each W is closed set in (N, σ) and $f^{-1}(W)$ is $closed$ set in (M, τ). Suppose that$(M, \tau)$is $open$ set in (N, σ) .

then W^c is closed set in (N, σ) By hypothesis $f^{-1}(W^c) = f^{-1}(W)^c$ is $closed$ set in (M, τ).

Hence $f^{-1}(W)$is open set in (M, τ). We get f is con. ■