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المرحلة : الرابعة

المادة: التبولوجيا العامة

عنوان المحاضرة: صفات التبولوجية في الفضاء التبولوجية

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We study topological and hereditary property of some separation

axioms.

Definition (34)

Let $f:(M,\tau) \to (N,\sigma)$ be any $Home$and δ any property in (M,τ) ,we say that topological property if δ is appear in (N,σ).

Proposition (35)

Let $f:(M,\tau) \to (N,\sigma)$be injective OM and if (M,τ) is a T_i_space, then $(N,\sigma)$is T_i_space, where $i = 0,1,2$.

Proof.

We prove that $i = 2$

Suppose that $\check{u} \neq \check{v} \in N$, since f is injective then $\exists\, u \neq v \in M \ni \check{u} = f(u)$ & $\check{v} = f\ (v)$. We get there exists H, D *are two disjoint* open sets in M such that $u \in H \wedge v \in D$ (because ((M,τ) is T_1_space) and $H \cap D = \emptyset$.

Since f is OM , then $f(H), f\ (D)$ are open sets of (M,τ) and $f(H \cap D) = \emptyset$, so $\check{u} = f(u) \in f(H)$ *and* $\check{v} = f(v) \in f\ (D)$.

Then (N,σ) is an T_2_space. In the some way prove i=0,1. ■

Proposition (36)

The property of a space being a T_i_ space is topological property. Where i =0,1,2.

Proof.

We prove that i=0

Suppose that $f:(M,\tau) \to (N,\sigma)$ is $Home_{N_P}$, $\check{u} \neq \check{v} \in M$.

Since f is bijective, then $\exists\, u, v \in M$such that $\check{u} = f(u)$, $\check{v}$= $f\ (v)$ and$u \neq v$.

Let (M,τ) be T_0__space for u *and* v , then $\exists\, H$ is open set $\ni$

$u \in H$, $v \notin H$, now $f\ (H)$is open set in (M,τ) (because H is *open* set in (M, τ) and f is OM),we get, $\check{u} \in f\ (H), \check{v}\ \notin f\ (H)$.

Hence $(N, \sigma)$is T_0_space.

In the some way prove i=1,2. ■

Remark (37)

A property δ of a topological space (M, τ) is said hereditary If and only if ∀subspace of (M, τ) also satisfies property δ.

Proposition (38)

The T_0_axiom of separation is hereditary property.

Proof.

We prove that i=0

Suppose that (M, τ) is T_0_space and $(N, \sigma)$is a subspace

on (M, τ) , As $u, v \in N \subseteq M$ if $u \neq v \in M$, we get there

exist open set on(M, τ) such that $u \in H\ , v \notin H$,

thus $\check{H} = \boldsymbol{N} \cap H \ \rightarrow\ \check{H}$ is $open$ set (because H is open set $\ni u \in H\ , v \notin H$), then $u \in \check{H}\ \&\ v \notin \check{H}$, hence (N, σ) is T_0 −space.■

To prove that $i = 2$

Suppose that (M, τ) is T_2_space and $(N, \sigma)$is a subspace on (M, τ) , for $u, v \in N \subseteq M$, $u \neq v \in M$, then $\exists\ H$, D are two disjoint open sets $\ni u \in H\ , v \notin H$ and $u \notin D\ , v \in D$, so $\check{H} = N \cap H$ and $\check{D} = N \cap D$.

Now $u \in \check{H} \in (N, \sigma)$and $v \in \check{D} \in (N, \sigma)$[because $u \in H \in (M, \tau)$ and $v \in D \in (M, \tau)$].

Since $H \cap D = \emptyset$, then $\check{H} \cap \check{D} = (N \cap H) \cap (N \cap D\ )$=

$N \cap (H \cap D) = N \cap \emptyset = \emptyset$.

Hence (N, σ) is T_2_space. In the some way prove i=1

Proposition

Let (M, τ) be compact space and (M, τ) be T_2 _space, $f: (M, \tau) \to (N, \sigma)$ is bijective and cont. , then $\mathfrak{f}$ is Hom.

Proof.

To prove that $\mathfrak{f}$ is Hom, it is enough to show that $\mathfrak{f}^{-1}$ is cont.

For this we must show that $\mathfrak{f}(F)$ is closed in N , for any $closed$ $F \subseteq \mathcal{M}$.Being a $clsoed$ subset of a compact set $\mathcal{M}$, F is compact set. then $\mathfrak{f}(F)$ is compact subset of T_2 −space $((N, \sigma)$and $\mathfrak{f}(F)$ is $closed$, for any $F \subseteq \mathcal{M}$ isclosed . which implies $\mathfrak{f}(F) \subseteq N$ is closed this $\mathfrak{f}^{-1}$ is cont.

Then $\mathfrak{f}$ is Hom because [$\mathfrak{f}$ are bijective, continuous and $\mathfrak{f}^{-1}$ is continuous].

Theorem

Let $f: (M, \tau) \to (N, \sigma)$be cont. and injective map, if (M, τ) compact space and $((N, \sigma)$ is T_2_space. Then $\mathcal{M}$ and N are homeomorphic.

Proof.

Let $\mathfrak{f}(\mathbb{A})$ be open in N , since $\mathcal{M}$ is compact, ∀ open set cover there corresponds a finite sub cover and N is T_2, for any $u \neq v \in \mathcal{M}$, ∃ two disjoint $\mathbb{H}$ and $\mathbb{D}$ are open set in N such that, $\mathfrak{f}(u) \in \mathbb{H}$, $\mathfrak{f}(v) \in \mathbb{D}$. Then open set $\mathbb{A}$ in $\mathcal{M}$, $\mathfrak{f}(\mathbb{A})$ is open set in N and $\mathcal{M} - \mathfrak{f}(\mathbb{A})$ is closed set in N.

Now to prove $\mathfrak{f}^{-1} = h: N \to \mathcal{M}$ is cont. . Also, to prove $h^{-1}(\mathbb{A})$ is open(closed) set in N .

Since $\mathbb{A}$ is open set and $\mathcal{M} - \mathbb{A}$ is closed set in $\mathcal{M}$, therefore $h^{-1}(N - \mathbb{A}) = \mathcal{M} - h^{-1}(\mathbb{A})$, we have $h^{-1}(N - \mathbb{A}) = \mathcal{M} - \mathfrak{f}(\mathbb{A})$--(i) [because $h^{-1} = \mathfrak{f}$].

From (i) for each closed set in $\mathcal{M}$, we get h^{-1} is closed set in N and h is cont. map. Therefore $\mathfrak{f}$ is Hom. , then $\mathcal{M}$ and N are homeomorphic. ■