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عنوان المحاضرة : تشاكل التبولوجي في الفضاء التبولوجية

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Definitions

A map $\mathfrak{f}:(M,\tau)\rightarrow(N,\sigma)$ is called :

1. $\mathcal{N}_{\mathcal{P}}$_ open map ($\mathcal{N}_{\mathcal{P}}$_OM) if, $\mathfrak{f}(\mathbb{A})\in\mathcal{N}_{\mathcal{P}}O(N,\sigma)$for each $\mathbb{A}\in\tau(\mathbb{X})$.

$\mathcal{N}_{\mathcal{P}}$_ closed map ($\mathcal{N}_{\mathcal{P}}$_CM) if, for each $\mathbb{A}^{c}\in\tau(\mathbb{X})$, $\mathfrak{f}(\mathbb{A}^{c})\in(N,\sigma)$.

Proposition

A bijective map $\mathfrak{f}:(M,\tau)\rightarrow(N,\sigma)$ is $\mathcal{N}_{\mathcal{P}}$_OM iff $\mathfrak{f}$ is $\mathcal{N}_{\mathcal{P}}$_CM

Proof.

Let $\mathfrak{f}$ be $\mathcal{N}_{\mathcal{P}}$_CM , bijective map and $\mathbb{A}\subseteq\mathcal{M}$,thus $\mathbb{A}^{c}$ is a $\mathcal{N}_{\mathcal{P}}$_ CS. Since $\mathfrak{f}$ is a $\mathcal{N}_{\mathcal{P}}$_CM, then $\mathfrak{f}(\mathbb{A}^{c})$ is a $\mathcal{N}_{\mathcal{P}}$_ CS in N.Therefore $\mathfrak{f}(\mathbb{A}^{c})^{c}$ is a $\mathcal{N}_{\mathcal{P}}$_ OS, so $\mathfrak{f}(\mathbb{A}^{c})^{c}=\mathfrak{f}(\mathbb{A})$ (because $\mathfrak{f}$ be bijective). Hence $\mathfrak{f}(\mathbb{A})$ is a $\mathcal{N}_{\mathcal{P}}$_ OS in N.

So $\mathfrak{f}$ is $\mathcal{N}_{\mathcal{P}}$_OM. In similar way we can prove that only if part. ■

Propositions

Let $\mathfrak{f}:(M,\tau)\rightarrow(N,\sigma)$ be a bijective map, then:

1. $\mathfrak{f}$ is a $\mathcal{N}_{\mathcal{P}}$_ OM iff $\mathfrak{f}^{-1}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$.
2. $\mathfrak{f}$ is a $\mathcal{N}_{\mathcal{P}}$_ CM iff $\mathfrak{f}^{-1}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$.

Proof.

1. Let $\mathfrak{f}$ is a $\mathcal{N}_{\mathcal{P}}$_OM and $\mathbb{A}\subseteq\mathcal{M}$, then $\mathfrak{f}(\mathbb{A})$ $\mathcal{N}_{\mathcal{P}}$_ OS in N, $(\mathfrak{f}^{-1})^{-1}(\mathbb{A})$= $\mathfrak{f}(\mathbb{A})$ $\mathcal{N}_{\mathcal{P}}$_ OS in $\breve{\mathcal{M}}$. Then $\mathfrak{f}^{-1}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$.

Conversely,

Let $\mathfrak{f}$ be bijective, then $\forall$ $\mathbb{A}\subseteq\mathcal{M}$,$(\mathfrak{f}^{-1})^{-1}(\mathbb{A})$= $\mathfrak{f}(\mathbb{A})$. Let $\mathbb{A}$ is $\mathcal{N}_{\mathcal{P}}$_OS in $\mathcal{M}$, we get $(\mathfrak{f}^{-1})^{-1}(\mathbb{A})$ is an $\mathcal{N}_{\mathcal{P}}$_ OS in N (because $\mathfrak{f}^{-1}$ is a $\mathrm{Con}_{\mathcal{N}_{\mathcal{P}}}$) and since $\mathfrak{f}$ be bijective, then $\mathfrak{f}(\mathbb{A})$is a $\mathcal{N}_{\mathcal{P}}$_ OS

In N. Hence $\mathfrak{f}$ is a $\mathcal{N}_{\mathcal{P}}$_ OM.

In similar way we can prove that (2). ■

Remark (4.2.7)

It is clear that every $Hom_{\mathcal{N}_{\mathcal{P}}}$ map is $\text{Con}_{\mathcal{N}_{\mathcal{P}}}$, but the converse is not true.

Because there exist the map $\mathfrak{f}$ is bijective, $\text{Con}_{\mathcal{N}_{\mathcal{P}}}$, but $\mathfrak{f}^{-1}$ not $\text{Con}_{\mathcal{N}_{\mathcal{P}}}$.

Therefore $\mathfrak{f}$ is not $Hom_{\mathcal{N}_{\mathcal{P}}}$.

Definition (14)

The bijective map $f: (M, \tau) \to (N, \sigma)$ is called homeomorphism

$(Home)$ if, it is Con .and OM.

Proposition (15)

The bijective map $f: (M, \tau) \to (N, \sigma)$ is a $Home$ if and only if $cl(f(A)) = f(cl(A))$, for each $A \subseteq M$.

Proof.

Let f be $Home$, therefore f be a Con and $closed$ function, so $f\ (cl(A)) \subseteq cl(f(A))$, $\forall A \subseteq M$. Since $cl(A)$ is closed set in $(M, \tau)$and f is closed, then $f(cl(A))$ is a CM in (N, σ), therefore $cl\left(f\left(cl(A)\right)\right) = f(cl_{N_P}(A))$ implies $cl(f(A)) \subseteq cl(f(cl(A))\ ) = f(cl(A))$ [because $A \subseteq cl(A), f(A) \subseteq f(cl(A))$], therefore $cl(f(A)) \subseteq f(cl(A))$. Then $cl\ (f(A)) = f\ (cl\ (A))$.

Conversely,

For each $A \subseteq M$ and $cl(f(A)) = f(cl(A)) \ni A = cl(A)$ then A is $closed$ set in (M, τ) and $f(A) = f(cl(A))$, so f is a Con. Therefore $f(A) = cl(f(A))$, thus $f(A)$ is a closed set, we get f is a CM and Con. Hence f is a $Home$■

Remarks (16)

1. If f is $Home$, then f^{-1} is also $Home$.

Since f is bijective, then f^{-1} is bijective , f is $Home$ and $f^{-1}$is Con. also $f = (f^{-1})^{-1}$ is Con. Therefore, $f^{-1}$is $Home$ map.

2. The bijective map $f: (M, \tau) \to (N, \sigma)$ is a $Home$ if and only if $cl\left(f^{-1}(B)\right) = f^{-1}(cl(B))$, $B \subseteq \acute{M}$.
3. The bijective map $f: (M, \tau) \to (N, \sigma)$ is a $Home$ if and only if $int\left(f^{-1}(B)\right) = f^{-1}(int\ (B))$, $B \subseteq \acute{M}$.

Propositions (17)

Let $f: (M, \tau) \to (N, \sigma)$ and $g: (M, \tau) \to (Y, \sigma)$

be two maps, then:

1. If f and g are CM (OM), then gof is CM (OM).
2. If gof is CM (OM) and f is surjective $Con.$ then g is CM (OM).
3. If gof is CM (OM)and g is surjective Con. then f is CM (OM).

Proof.

1. For each D be closed set in (M, τ) then $f(D)$ is a closed set in $\left(\acute{M}, I(\acute{X})\right)$,thus $g\left(f(D)\right)$ is $closed$ set in (Y, σ) . But, $gof(D) = g(f(D))$. Hence gof is CM. ■
2. For each D is closed set in (Y, σ) then $f^{-1}(D)$ is closed set in (M, τ)thus $gof(f^{-1}(D))$ is closed set in (Y, σ) Since f is onto then $gof\left(f^{-1}(D)\right) = g(D)$, hence $g(D)$is closed set in (N, σ). Thus g is CM. ■

3. For each D is closed set in (M,τ) and let $gof(D)$ be closed in $\left(\bar{M}, I(\bar{X})\right)$, we get $g^{-1}\left(gof(D)\right)$ is closed in (N,σ)Hence $g^{-1}\left(gof(D)\right) = f(D)$(because g is onto), so $f(D)$ is closed set in (N,σ), then f is CM. ■

Remark (18)

The two (M,τ) $and(N,\sigma)$ are said to be homeomorphic if there exists $Home$ from (M,τ) to $(N,\sigma)$and denoted by $(M,\tau) \cong (N,\sigma)$.